

## The Femtoscope: The Missing Link between Nuclear Physics and Game Theory

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### 1. Abstract

The Femtoscope explains the holographic principle and the alpha decay through the Navier Stokes equations. The surface of the atomic nucleus stores all the codifiable information of the incompressible nuclear fluid; it is constituted exclusively by Z protons in equilibrium with the N neutrons of the inner sphere. This equilibrium creates the nuclear viscosity and is represented by the conservation of angular momentum at the boundary of the proton layer and the neutron sphere. We demonstrate two outstanding facts. That the nuclear cavitation produced by the atypical variation of the nuclear pressure is responsible for the formation of alpha particles for the 670 isotopes with this decay and that the atomic nuclei of Cr, Tu and Xe detect dark matter. In addition, the minimum entropy manages the dynamics of the formation and emission of helium nuclei that maintain nuclear stability, rebalancing the variation of nuclear pressure. Finally, the dominant force in the alpha decay is neither the nuclear nor the electric decay but the force of Navier Stokes, which explains why the alpha decays, might not be produced by tunneling. The holographic principle is fundamental in the superstrings and quantum gravity theories (Gerard t Hooft and Leonard Susskind). It postulates that all the information contained in a certain volume of a finite space can be known from the codifiable information that remains in the border of said region. The Bekenstein border or upper limit to the entropy, is

the maximum amount of information necessary to perfectly describe a physical system up to the quantum level[1-3]. The Navier Stokes equations are a problem of the millennium that has not been resolved yet in a generalized manner. We present a particular solution that logically meets all the requirements established by the Clay Foundation. This solution coherently explains the incompressible nuclear fluid and allows calculations of the nuclear viscosity and nuclear pressure that are key elements of nuclear cavitations. Theory of Quantum Games. Widely used and accepted in its mathematical formalization and in its applications in Information Theory [4,5]. The alpha particle is one of the most stable. Therefore it is believed that it can exist as such in the heavy core structure. The kinetic energy typical of the alpha particles resulting from the decay is in the order of 5 MeV. Its speed is 15,000 km/s. Applying the holographic principle we can explain formation and decay of these helium nucleus [6].

The study of the atomic nucleus has been successful through two well structured visions (layer model and drop model), which however leave some unexplained topics, such as. Alpha emissions, nuclear viscosity, angular moments and the circular trajectories of the nucleons. Could it seem that there is maximum entropy in its structure and geometry? [3,6,7]. The layer model tries to capture part of the internal structure reflected in its angular and spin moment; it tries to explain why the nucleus with a "magic number" of nucleons turn out to be more

stable. The explanation of the model is that the nucleons are grouped into "layers". In an abstract way, each layer is formed by a set of quantum states with similar energies. The drop model in an analogy with a mass of fluid, tries to describe both the bond energies between neutrons and protons and some aspects of the excited states of an atomic nucleus, which are reflected in the nuclear spectrums. We will present nine theoretical and/or experimental evidences that show that the holographic principle is fulfilled in the atomic nucleus and that the alpha decay is a phenomenon of nuclear cavitations [6, 8].

The atomic nucleus is formed by two layers, an internal one populated by N neutrons and an external one constituted by Z protons that rotate collectively around the neutrons, defining a nuclear viscosity.

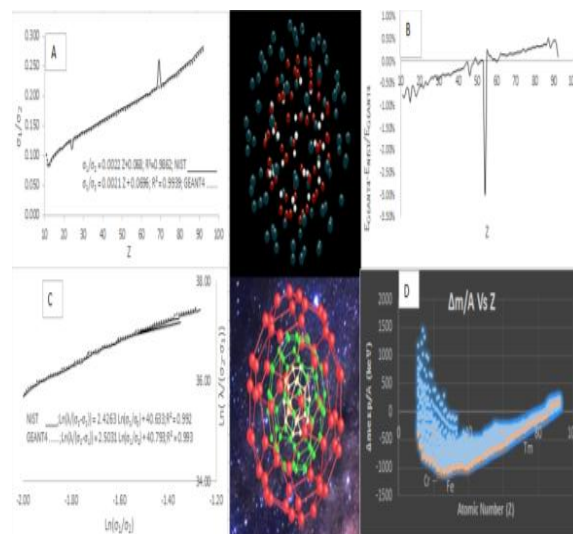
- The incompressible Fluid model integrates the two previous models, because, on the one hand, it predicts the existence of two totally defined spherical trajectories. That of neutrons formed by a sphere with a radius  $0 \leq R \leq 1.2 N^{1/3}$  and the proton layer with a radius  $1.2(N)^{1/3} \leq R \leq 1.2(N+Z)^{1/3}$ . The nuclear stability evaluated for all the atoms and isotopes, allows us to find the radius of the proton  $r_p \leq 1.2(N+Z)^{1/3} - 1.2(N)^{1/3}$ .
- The only way to place Z protons at maximum distance between them, but at a minimum distance (contact) with the neutrons, is to collectively rotate around the neutron sphere. This architecture of the nucleus, respects the Pauli Exclusion Principle. In this way, the distance between proton-proton is maximum the distance between proton-neutron is minimal, and the distance between neutron-neutron is minimal.
- The spins of the protons and neutrons are paired separately, exclusively protons with protons and neutrons with neutrons. Indicating that there are two totally defined zones of protons and neutrons.
- The Classic Physics is still valid. Since, the angular moments of protons  $L_p$  and neutrons  $L_N$  are the same in the basic state:  
$$L = I_p \omega_p = I_N \omega_N$$
- There is a nuclear viscosity between the neutron sphere and the proton layer, which rotate one respect to each other.
- A system with an ordered and optimal structure must have minimum entropy.

- The formation of alpha particles is the result of nuclear cavitation.
- The emission of alpha particles is a result of a cooperative equilibrium between protons and neutrons.
- All the atomic nucleus of the isotopes comply with the Nash equilibrium in mixed strategy, which is of minimum entropy.

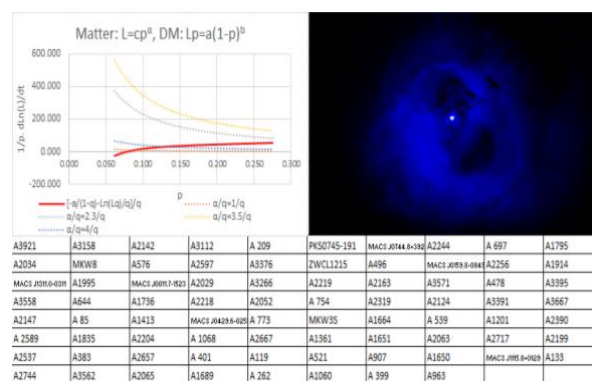
The holographic principle allows obtaining the entropy of a black holes, and quantum systems. For the atomic nucleus it is demonstrated that the entropy value is minimum, which fits the game theory. Until now, dark matter is totally invisible and hidden for us, it does not look like anything seen and studied on planet earth. The standard model of cosmology indicates that the total mass energy of the universe contains 4.9% ordinary matter, 26.8% dark matter and 68.3% dark energy [9-11]. The most widely accepted hypothesis on the form for dark matter is that it is composed of weakly interacting massive particles, WIMPs that interact only through gravity and the weak force. Because WIMPs are so "weakly interacting" that is, they cannot interact with normal matter via the electromagnetic, strong or weak forces [9], says main researchers of dark matter such us. NASA's Chandra, the SNOLAB underground laboratory at Sudbury, the Gran Sasso National Laboratory, the Canfranc Underground Laboratory, the Boulby Underground Laboratory, the deep underground science and engineering laboratory, the China Jinping Underground Laboratory and Alpha Magnetic Spectrometer (AMS) [10]. In the AMS, direct evidence of the collision of two dark matter particles is sought. Saul Perlmutter, Nobel Prize in Physics 2011, showed that the expansion of the universe is accelerating, and matter and dark energy play an important role.

The direct detection of Dark Matter can take place through their interaction with nuclei inside a detector. The nuclear recoiling energy need to be measured or secondary effects such us. Ionization on solids, Ionization in scintillators (measured by the emitted photons) or Temperature increase (measured by the released phonons). Direct methods seek the unmistakable revelation of dark matter and the measurement of some physical parameter such as the cross section or energy; these are performed in particle accelerators and in underground tunnels. With these methods we also try to create dark matter and study its properties [11-13]. Indirect methods seek the measurement of physical parameters in secondary phenomena produced by the action and presence of dark matter. In addition, direct or indirect

effects are sought in the environment surrounding dark matter [11,13]. One of the results that agree between research with X-ray Femtoscope and X-ray telescope has to do with low-energy x-rays, which have aroused interest in several laboratories including the NASA's Chandra X-ray Observatory, the Japanese-led X-ray telescope known as Hitomi and the European Space Agency's (ESA) XMM-Newton, textually it is said "Mysterious X-Ray Emission May Reveal Nature of Dark Matter, researchers have found it nearly impossible to explain why that particular wavelength (3.5KeV) is being emitted and absorbed, and because previously-seen astronomical objects don't seem to reveal clues about the wavelength emission, they suggested dark matter could be the cause" [10]. It has been accepted to consider the atomic nucleus as an incompressible fluid and nevertheless there is no attempt to prove it neither experimentally nor theoretically. During the last century some theoretical models were proposed in terms of the nuclear viscosity by using Navier Stokes equations but they did not answer the question of nuclear incompressibility [14,15]. The nuclear stability and surface structure have been experimentally studied using scattering theory but many questions still need to be answered such as. What happened to the nuclear surface during nuclear disintegration and what is its influence inside the nuclear volume? The nuclear surface and volume can be correlated if we consider the nucleus as incompressible fluid. On this way, anything that happens at the surface can be discovered inside the nucleus and vice-versa. It is similar to action and reaction law, [16-18]. Femtoscope measurements are related to atomic K-edge resonances which in turn are related to nucleon cross sections at nuclear surface. This last, allow us the calculation of several parameters such as nuclear viscosity or nuclear force inside the nucleus. In summary, the Navier Stokes equations are the bridge between physical processes at the nucleus surface and inside of it (Figures 1, 2 and Table1).



**Figure 1:** Cross section excess measure presence of dark matter for  $^{24}\text{Cr}$  and  $^{69}\text{Tm}$ . B.-Evolution of nuclear stability P, depending on the ratio of the cross sections. We obtained the probability P (x, y, z, t) which is the fundamental solution of the Navier Stokes equations, measures the Nuclear stability and represents both the behavior of the atomic nucleus in equilibrium and out of equilibrium such as gamma or beta decay C.- Calculation of the Rydberg constant using Navier Stokes model. D-Mass excess. For Cr, we can see the resonance in mass.



**Figure 2:** From the information of 81 galaxy clusters, it is observed that the logarithmic derivative of the luminosity causes the dark matter to behave as a catalyst or as an inhibitor of the nuclear reaction, (RED).

ELE MEN T	Z	N	A	$E_2=$ $E_1=$ E	$\lambda_{Rx}$	$\sigma_2$	$\sigma_1$	$(\sigma_2-$ $\sigma_1)/$ $\sigma_1$	$\sigma_1/\sigma_2$	Estability: P	$\varepsilon_{Z/N}$	Y	x
			Me V	m	m2	m2	%	GEA NT4	1- $(\sigma_1/\sigma_2)2.5$ 031	%	$\ln(\lambda/(\sigma_2-$ $\sigma_1))$	$\ln(\sigma_1/$ $\sigma_2)$	
Na	11	1 2	2 3	1.06 E-03	1.165 3E-09	9.0 6E-25	9.1 8E-26	888 %	0.101	0.997	0.0 009	34.89 7	(2.290 1)
Mg	12	1 2	2 4	1.29 E-03	9.577 8E-10	9.3 7E-25	7.6 6E-26	112 2%	0.082	0.998	0.0 007	34.64 7	(2.503 2)
Al	13	1 4	2 7	1.55 E-03	7.999 5E-10	7.0 3E-25	6.1 0E-26	105 3%	0.087	0.998	0.0 009	34.75 8	(2.445 0)
Si	14	1 4	2 8	1.83 E-03	6.780 7E-10	5.5 0E-25	5.2 1E-26	956 %	0.095	0.997	0.0 007	34.84 8	(2.357 5)
P	15	1 6	3 1	2.13 E-03	5.819 8E-10	4.2 8E-25	4.2 2E-26	916 %	0.098	0.997	0.0 009	34.94 9	(2.318 1)
S	16	1 6	3 2	2.46 E-03	5.048 4E-10	3.5 5E-25	3.6 4E-26	877 %	0.102	0.997	0.0 007	34.99 8	(2.278 8)
Cl	17	1 8	3 5	2.80 E-03	4.420 3E-10	2.8 0E-25	2.9 9E-26	837 %	0.107	0.996	0.0 008	35.10 9	(2.237 5)
Ar	18	1 8	3 6	3.18 E-03	3.901 8E-10	2.1 9E-25	2.4 0E-26	810 %	0.110	0.996	0.0 007	35.23 4	(2.208 7)
K	19	2 0	3 9	3.58 E-03	3.460 1E-10	2.0 0E-25	2.2 4E-26	796 %	0.112	0.996	0.0 008	35.20 3	(2.192 3)
Ca	20	2 0	4 0	4.02 E-03	3.088 0E-10	1.7 2E-25	1.9 9E-26	763 %	0.116	0.995	0.0 007	35.24 9	(2.155 8)
Sc	21	2 4	4 5	4.47 E-03	2.776 3E-10	1.3 8E-25	1.6 3E-26	747 %	0.118	0.995	0.0 010	35.36 3	(2.136 9)
Ti	22	2 6	4 8	4.94 E-03	2.509 5E-10	1.1 6E-25	1.4 0E-26	726 %	0.121	0.995	0.0 011	35.44 1	(2.112 0)
V	23	2 8	5 1	5.44 E-03	2.279 3E-10	9.8 8E-26	1.2 2E-26	712 %	0.123	0.995	0.0 012	35.50 6	(2.094 4)
Cr	24	2 8	5 2	5.96 E-03	2.081 1E-10	8.7 3E-26	1.1 0E-26	695 %	0.126	0.994	0.0 011	35.54 2	(2.072 9)
Mn	25	3 0	5 5	6.51 E-03	1.904 3E-10	7.6 3E-26	9.6 5E-27	691 %	0.126	0.994	0.0 011	35.58 8	(2.067 9)
Fe	26	3 0	5 6	7.08 E-03	1.750 3E-10	6.9 0E-26	8.8 7E-27	678 %	0.129	0.994	0.0 010	35.60 8	(2.051 2)
Co	27	3 4	5 7	7.68 E-03	1.614 3E-10	5.9 0E-26	7.8 7E-27	663 %	0.131	0.994	0.0 010	35.67 8	(2.032 2)

		2	9	E-	2E-10	7E-	3E-	%			011	3	2)
				03		26	27						
<b>Ni</b>	28	3	5	8.30	1.493	5.5	7.3	656	0.132	0.994	0.0	35.66	(2.023
		0	8	E-03	3E-10	6E-	5E-	%			009	9	1)
						26	27						
<b>Cu</b>	29	3	6	8.94	1.386	4.6	6.3	631	0.137	0.993	0.0	35.77	(1.989
		4	3	E-03	4E-10	7E-	9E-	%			011	3	8)
						26	27						
<b>Zn</b>	30	3	6	9.62	1.288	4.2	5.8	631	0.137	0.993	0.0	35.79	(1.989
		4	4	E-03	5E-10	7E-	3E-	%			010	1	6)
						26	27						
<b>Ga</b>	31	3	6	1.03	1.200	3.7	5.1	623	0.138	0.993	0.0	35.86	(1.978
		8	9	E-02	1E-10	1E-	3E-	%			012	2	5)
						26	27						
<b>Ge</b>	32	4	7	1.11	1.120	3.3	4.6	612	0.140	0.993	0.0	35.90	(1.962
		1	3	E-02	3E-10	3E-	7E-	%			013	5	9)
						26	27						
<b>As</b>	33	4	7	1.18	1.048	3.0	4.2	600	0.143	0.992	0.0	35.94	(1.945
		2	5	E-02	1E-10	0E-	9E-	%			013	4	5)
						26	27						
<b>Se</b>	34	4	7	1.26	9.825	2.6	3.8	598	0.143	0.992	0.0	36.00	(1.943
		4	8	E-02	2E-11	5E-	0E-	%			014	3	0)
						26	27						
<b>Br</b>	35	4	7	1.34	9.228	2.4	3.6	581	0.147	0.992	0.0	36.02	(1.918
		4	9	E-02	4E-11	6E-	1E-	%			013	1	7)
						26	27						
<b>Kr</b>	36	4	8	1.43	8.682	2.2	3.2	569	0.149	0.991	0.0	36.07	(1.901
		8	4	E-02	3E-11	0E-	9E-	%			014	4	1)
						26	27						
<b>Rb</b>	37	4	8	1.52	8.178	2.0	3.0	564	0.151	0.991	0.0	36.09	(1.893
		8	5	E-02	9E-11	3E-	5E-	%			014	8	4)
						26	27						
<b>Sr</b>	38	5	8	1.61	7.716	1.8	2.8	558	0.152	0.991	0.0	36.13	(1.884
		0	8	E-	7E-11	5E-	1E-	%			014	2	1)
				02		26	27						
<b>Y</b>	39	5	8	1.70	7.293	1.7	2.6	549	0.154	0.991	0.0	36.14	(1.870
		0	9	E-02	2E-11	3E-	6E-	%			013	7	2)
						26	27						
<b>Zr</b>	40	5	9	1.80	6.902	1.5	2.4	537	0.157	0.990	0.0	36.18	(1.851
		1	1	E-02	6E-11	8E-	9E-	%			013	1	9)
						26	27						
<b>Nb</b>	41	5	9	1.89	6.543	1.4	2.3	532	0.158	0.990	0.0	36.20	(1.844
		3	4	E-02	4E-11	7E-	3E-	%			014	1	2)
						26	27						
<b>Mo</b>	42	5	9	2.00	6.210	1.3	2.1	529	0.159	0.990	0.0	36.24	(1.839
		4	6	E-02	1E-11	4E-	2E-	%			013	8	6)
						26	27						
<b>Tc</b>	43	5	9	2.10	5.900	1.2	2.0	518	0.162	0.990	0.0	36.26	(1.822
		5	8	E-	7E-11	5E-	2E-	%			013	9	0)
				02		26	27						
<b>Ru</b>	44	5	1	2.21	5.613	1.1	1.8	509	0.164	0.989	0.0	36.30	(1.806
		7	0	E-02	4E-11	5E-	9E-	%			014	5	2)
			1			26	27						
<b>Rh</b>	45	5	1	2.32	5.346	1.0	1.7	501	0.166	0.989	0.0	36.32	(1.793
		8	0	E-02	2E-11	7E-	8E-	%			013	9	6)

<b>Pd</b>	46	6	1	2.43	5.097	9.8	1.6	496	0.168	0.989	0.0	36.37	(1.784
	0	0	0	E-02	8E-11	0E-	4E-	%			014	2	6)
<b>Ag</b>	47	6	1	2.55	4.864	9.2	1.5	496	0.168	0.989	0.0	36.38	(1.785
	0	0	0	E-02	2E-11	2E-	5E-	%			013	6	8)
<b>Cd</b>	48	6	1	2.67	4.645	8.4	1.4	482	0.172	0.988	0.0	36.43	(1.760
	3	1	1	E-02	7E-11	2E-	5E-	%			014	6	8)
<b>In</b>	49	6	1	2.79	4.440	7.8	1.3	474	0.174	0.987	0.0	36.46	(1.747
	4	1	1	E-02	5E-11	6E-	7E-	%			014	2	2)
<b>Sn</b>	50	6	1	2.92	4.248	7.2	1.2	467	0.176	0.987	0.0	36.50	(1.734
	9	1	1	E-02	4E-11	4E-	8E-	%			016	3	6)
<b>Sb</b>	51	7	1	3.05	4.067	6.7	1.2	464	0.177	0.987	0.0	36.53	(1.730
	0	2	1	E-02	8E-11	5E-	0E-	%			015	0	3)
<b>Te</b>	52	7	1	3.18	3.898	6.1	1.0	466	0.177	0.987	0.0	36.57	(1.733
	6	2	8	E-02	3E-11	6E-	9E-	%			018	9	7)
<b>I</b>	53	7	1	3.32	3.738	5.9	1.0	454	0.181	0.986	0.0	36.58	(1.711
	4	2	7	E-02	5E-11	3E-	7E-	%			016	0	8)
<b>Xe</b>	54	7	1	3.46	3.587	5.5	1.0	449	0.182	0.986	0.0	36.61	(1.702
	7	3	1	E-	9E-11	1E-	0E-	%			017	4	2)
<b>Cs</b>	55	7	1	3.60	3.445	5.1	9.6	438	0.186	0.985	0.0	36.63	(1.682
	8	3	3	E-02	4E-11	9E-	4E-	%			017	8	8)
<b>Ba</b>	56	8	1	3.74	3.310	4.8	9.0	433	0.188	0.985	0.0	36.67	(1.673
	1	3	7	E-02	8E-11	1E-	2E-	%			017	6	4)
<b>La</b>	57	8	1	3.89	3.183	4.5	8.5	432	0.188	0.985	0.0	36.69	(1.672
	2	3	9	E-	7E-11	5E-	5E-	%			017	2	1)
<b>Ce</b>	58	8	1	4.05	3.064	4.3	8.1	429	0.189	0.985	0.0	36.70	(1.666
	2	4	0	E-02	7E-11	2E-	6E-	%			016	7	3)
<b>Pr</b>	59	8	1	4.20	2.951	4.1	7.9	418	0.193	0.984	0.0	36.71	(1.645
	2	4	1	E-02	3E-11	4E-	8E-	%			016	8	8)
<b>Nd</b>	60	8	1	4.36	2.843	3.8	7.5	412	0.195	0.983	0.0	36.74	(1.632
	3	4	3	E-02	9E-11	8E-	9E-	%			016	7	2)
<b>Pm</b>	61	8	1	4.52	2.741	3.7	7.3	406	0.198	0.983	0.0	36.75	(1.621
	4	4	5	E-02	9E-11	2E-	5E-	%			016	6	8)
<b>Sm</b>	62	8	1	4.69	2.644	3.4	6.8	401	0.199	0.982	0.0	36.79	(1.612
	7	4	9	E-02	9E-11	5E-	8E-	%			016	9	0)
<b>Eu</b>	63	8	1	4.86	2.552	3.2	6.5	398	0.201	0.982	0.0	36.81	(1.604
	8	5	1	E-02	8E-11	8E-	9E-	%			016	5	9)

<b>Gd</b>	64	9	1	5.03	2.464	3.0	6.0	400	0.200	0.982	0.0	36.85	(1.609
		3	5	E-02	7E-11	5E-	9E-	%			017	2	4)
			7			27	28						
<b>Tb</b>	65	9	1	5.21	2.381	2.9	5.9	390	0.204	0.981	0.0	36.87	(1.589
		4	5	E-02	6E-11	1E-	4E-	%			017	0	0)
			9			27	28						
<b>Dy</b>	66	9	1	5.39	2.301	2.7	5.7	381	0.208	0.980	0.0	36.89	(1.571
		7	6	E-02	9E-11	5E-	1E-	%			018	8	0)
			3			27	28						
<b>Ho</b>	67	9	1	5.57	2.226	2.6	5.4	375	0.210	0.980	0.0	36.91	(1.558
		8	6	E-02	0E-11	1E-	9E-	%			018	8	8)
			5			27	28						
<b>Er</b>	68	9	1	5.76	2.153	2.4	5.2	371	0.213	0.979	0.0	36.93	(1.548
		9	6	E-	5E-11	9E-	8E-	%			017	7	7)
			7	02		27	28						
<b>Tm</b>	69	1	1	5.95	2.084	2.3	5.0	367	0.214	0.979	0.0	36.95	(1.540
		0	6	E-02	2E-11	7E-	8E-	%			017	3	6)
			0	9		27	28						

**Table 1:** Data and Information of Geant4 and NIST. Resonance energies of each of the atoms of the periodic table.

The Yukawa model gives us information about the nuclear potential and it has been successfully proven in nuclear reactions, electricity production through nuclear energy and also allowed the advance of Nuclear Theory [18,19]. The spatial Fermi Dirac distribution allowed us to find the nuclear radius as a function of the Atomic Mass  $A$ , namely,  $R = 1.2 A^{1/3}$ , but it is unable to distinguish protons from neutrons. The following question arises, if the proton and neutron masses are different, what happen to their respective radii? The structure of condensed matter or atoms can be studied through scattering of X-rays, neutrons, and electrons. We are interested in the low-energy region where the nucleons hardly get exited internally. In this energy region we can treat the nucleons as inert, structure less elementary particles and we can study nucleon-WIMPs interactions. We also consider the nucleons as non relativistic so their interactions can be described by a potential. The approach we use is phenomenological since we extract the nucleon interactions from atomic K-edge data and, by applying Navier Stokes equations to nuclear fluid, considered as incompressible, we made predictions for the nuclear many-body system and dark matter detector [9,10,15]. In light of modern quantum theory, the boundary of compact support of the mass distribution of a particle, just like the location of an proton, neutron or electron cannot be specified precisely. However, it is still advantageous to conceptualize an atomic system in classical terms, especially for specific cross section environments wherein best-fit classical parameters are measured (NS). The electrons in the atomic K-shell orbit much

closer to the nucleus than other shells. At low energy (up to 0.116 MeV), it is well known that the photoelectric effect is more important than other effects for most elements on the periodic table. We can see different peaks of photoelectric absorption being the strongest ones due to electrons located in the K-shell. Somehow, the system formed by electron, photon and nucleus interacts strongly in the K-shell when compared to other shells. The data base at the National Institute of Standards and Technology (NIST) and GEANT4 simulation contain the tabulated photoelectric peak cross sections (K edge) for every element on the periodic table [7,8,19].

## 2. Model

### 2.1. The Atomic Nucleus Is a Cooperative System of Minimum Entropy

Let  $\Gamma = (I, S, v)$  represent a finite game in strategy form, with  $I$  as the set of players (protons and neutrons) of cardinality  $n \in N$ , then every player is noted  $n \in I$ . The finite set  $S_n$  of cardinality  $m_n \in N$  is the set of pure strategies of each player  $n \in I$ ,  $s_{n_j} \in S_n$ ,  $j = 1, \dots, m_n$  and  $S = \prod_{n \in I} S_n$  designates the set of profiles in pure strategies of the game with  $s \in S$  an element of that set. The function  $v : S \rightarrow R^n$  associates every profile  $s \in S$  the vector of utilities  $v(s) = (v_1(s), \dots, v_n(s))$ , where  $v_n(s)$  designates the utility of the player  $n$  facing the

profile  $s$ . If the mixed strategies are allowed then we have.

$$\Delta(S_n) = \left\{ p_n \in R^{m_n} : \sum_{j \in S_n} p_{nj} = 1 \right\}$$

the unit simplex of the mixed strategies of player  $n \in I$ . We will note  $p_n = (p_{nj})_j \in S_n$ . The set of profiles in mixed strategies is the polyhedron  $\Delta$  with  $\Delta = \prod_{n \in I} \Delta(S_n)$  and  $p \in \Delta$  one point of  $\Delta$ , where  $p = (p_1, \dots, p_n)$ , and  $p_n = (p_{n1}, p_{n2}, \dots)$ .

The function  $u : \Delta \rightarrow R_+^n$  associates with every profile in mixed strategies  $p \in \Delta$  the vector of expected utilities  $\bar{u}(p) = (\bar{u}_1(p, s_{1j}), \dots, \bar{u}_n(p, s_{nj}), \dots)$ , where  $\bar{u}_n(p)$  is the expected utility of the player  $n \in I$ .

Every  $(u_1(1, p_{-n}, s_{nj}), \dots, u_n(m_n, p_{-n}, s_{nj}))$  represents the player's preferences  $n \in I$ . The triplet  $(I, \Delta, \bar{u})$  designates the extension of the game  $\Gamma$  to the mixed strategies. If the function of payments is defined by  $u_i(p) = \sum_{s \in S} p(s)u_i(s)$ , where  $p(s) = \prod_{i \in I} p_i(s_i)$  and if  $p = (p_1, \dots, p_n) \in \Delta$ , then we get Nash's equilibrium if, and only if,  $\forall n \in I$ , and  $\forall p_n \in \Delta(S_n)$ , the next inequality is respected  $\bar{u}_n(p^*) \geq \bar{u}_n(p_n, p_{-n}^*)$ . The sets of  $Z$  protons and  $N$  neutrons are the players who interact strategically in pairs through the potential of Yukawa and the forces of nature. The fundamental interactions occur through the orbital, the spin and the Navier Stokes forces.

- **Theorem 1:** The game entropy is minimum only in Nash's equilibrium  $(\min(H_i)) \Leftrightarrow (\sigma_i(\lambda) = 0)$ .
- **Proof:** In the Nash equilibrium, the utility function obeys the following properties.

$$u_i(j, p_{-i}) = \bar{u}_i(p), \forall i \in I, \forall j = 1, \dots, m_i$$

$$\bar{u}_i(p) = \sum_{j=1}^{m_i} p_{ij} \sum_{k_1=1}^{m_i} \dots \sum_{k_n=1}^{m_i} (p_i k_1 \dots p_i k_n) u_i(j, k_1, \dots, k_n)$$

$$\bar{u}_i(p) = \sum_{j=1}^{m_i} p_{ij} u_i(j, p_{-i})$$

$$\sigma_i^2(\lambda) = \sum_{j=1}^{m_i} (u_i(j, p_{-i}) - \bar{u}_i)^2 p_{ij} = 0$$

Outside of Nash's equilibrium, we can measure the standard deviation:  $\sigma_i^2(\lambda) \neq 0$ . If the probability density of a variable  $X$  is normal:  $N(\mu, \sigma)$ , then its entropy is minimum for the null standard deviation  $\sigma_i = 0$ .

$$S = - \int_{-\infty}^{\infty} p(x) \ln(p(x)) dx = - \int_{-\infty}^{\infty} p(x) \left[ \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{(x-\mu)^2}{2\sigma^2} \right] dx$$

$$S = \ln(\sqrt{2\pi\sigma}) + \frac{\sigma^2}{2} = (1 + \ln \sqrt{2\pi}) + \ln \sigma$$

If the probability functions of:  $u_i(j, p_{-i})$  is a multinomial log of parameter  $\lambda$ , then its entropy is minimum and its standard deviation is null for  $\lambda \rightarrow \infty$ .

The nature of the barium matter has the following structure of isotopes, for a total of 3179, <https://www.nndc.bnl.gov/nudat2/> and we can represent it as:

Z / N	odd	even
odd	749	782
even	816	832

In this game the probability of having 749 isotopes, with odd-protons and odd-neutrons is  $(p = 0.4815, q = 0.4922)$ . Giving us an equiprobable scheme, or maximum entropy.

$$S = \frac{749}{3179} \ln\left(\frac{3179}{749}\right) + \frac{782}{3179} \ln\left(\frac{3179}{782}\right) + \frac{816}{3179} \ln\left(\frac{3179}{816}\right) + \frac{832}{3179} \ln\left(\frac{3179}{832}\right) = 1.3855$$



For the case of isotopes with alpha decay we have a total of 371. Given the next game in the matrix form for the proton and neutron spins represented by

$$\left( \begin{array}{|c|c|c|} \hline Z / N & odd & even \\ \hline odd & 60 & 52 \\ \hline even & 61 & 198 \\ \hline \end{array} \right), \text{ which have an}$$

experimental probability structure of ( $p = 0.3018, q = 0.3261$ ). While the probability values depend on the spin matrix

$$\left( \begin{array}{|c|c|c|} \hline Z / N & odd : q & even : 1 - q \\ \hline odd : p & -1 & + \frac{1}{2} \\ \hline even : 1 - q & + \frac{1}{2} & 0 \\ \hline \end{array} \right) \text{ is}$$

( $p = 1/4, q = 3/4$ ) that maintain an appropriate correspondence, showing that the universe is built in an optimal way, and with minimum entropy

$$S = \frac{60}{371} \ln\left(\frac{371}{60}\right) + \frac{52}{371} \ln\left(\frac{371}{52}\right) + \frac{61}{371} \ln\left(\frac{371}{61}\right) + \frac{198}{371} \ln\left(\frac{371}{198}\right) = 1.202$$

- **Theorem 2:** The energy of an alpha particle  $Q_\alpha = S_n^a S_p^b$ , is a cooperative balance of neutron energy  $S_n > 0$  and the proton energy  $S_p > 0$  where:  $0 < a < 1, 0 < b < 1, a + b < 1$ .

- **Proof:** The Hessian of  $Q_\alpha$  have the form

$$\left( \begin{array}{|c|c|} \hline a(a-1)S_n^{a-2}S_p^b & abS_n^{a-1}S_p^{b-1} \\ \hline abS_n^{a-1}S_p^{b-1} & b(b-1)S_n^a S_p^{b-2} \\ \hline \end{array} \right)$$

The determinant of the Hessian determines that the function  $Q$  is convex for

$ab(1-a-b)S_n^{a-2}S_p^{b-2} > 0$  and is concave for  $ab(1-a-b)S_n^{a-2}S_p^{b-2} < 0$ . In this way protons and neutrons act cooperatively.

This theorem is determined by the joint action of protons and neutrons. Therefore, the energy of the alpha particles is determined by the energy of the protons and neutrons emitted.

## 2.2. Spherical Trajectory of Nucleon Layers

- **Theorem 3:** An action on the nuclear surface produces a reaction in the nuclear volume and vice versa.

- **Proof:** The volume and the nuclear surface are connected through the Gaussian divergence theorem and the Navier Stokes equations. For an incompressible Fluid, whose velocity field  $\vec{u}(x, y, z)$  is given,  $\vec{\nabla} \cdot \vec{u} = 0$  is fulfilled. Logically, the integral of this term remains zero, that is:

$$\int \int \int \vec{\nabla} \cdot \vec{u} dx dy dz = 0$$

Writing the Divergence theorem.  $\int \int \vec{u} \cdot \vec{n} dS = \int \int \int \vec{\nabla} \cdot \vec{u} dx dy dz = 0$ , the first term must be equal to zero, that is:

$$\int \int \int \vec{u} \cdot \vec{n} dS = \int \int \|\vec{u}\| \|\vec{n}\| \cos(\alpha) dS = 0 \rightarrow \alpha = \frac{\pi}{2}$$

The only possible trajectory is circular, because in this case the vector  $\vec{n}$  is perpendicular to the surface of the sphere. In this way the equation of the outer sphere corresponding to the surface is:  $x^2 + y^2 + z^2 = 1.2 A^{1/3}$ . Within the nuclear Fluid there are layers of nucleons that move in spherical trajectories.

- **Remark 4:** The information in the volume of the inner layer that contains the neutrons  $\int \int \int \vec{\nabla} \cdot \vec{u} dx dy dz$  is equal to the information of the outer layer that contains the protons  $\int \int \int \vec{u} \cdot \vec{n} dS$ .

- **Theorem 5:** In the atomic nucleus, the angular moments of the neutron sphere and the proton layer are canceled  $I_n w_n - I_p w_p = 0$ .

The angular momentum in a classical way is given by  $L = I_n w_n - I_p w_p$ , where

$$I_n = \frac{2}{5} (1.2)^2 N^{5/3} m_n \text{ is the angular momentum of}$$

the neutron sphere with its respective angular velocity  $w_n$  while for protons we have

$$I_p = \frac{2}{5} m_p (1.2)^2 [(N+Z)^{5/3} - N^{5/3}]. \text{ Clearing}$$

$w_n/w_p$  of the equation  $I_n w_n - I_p w_p = 0$  you get a theoretical result given by

$$\frac{w_n}{w_p} = \left( \left( \frac{N+Z}{N} \right)^{5/3} - 1 \right) \frac{m_p}{m_n}$$

It is easy to obtain an experimental result based on the measured energies of the protons and neutrons that leave the atomic nucleus, where:

$$\frac{w_n}{w_p} = \sqrt{\frac{S_n m_p}{S_p m_n}}$$

In this relation  $w_n/w_p$  we take into consideration that the protons and neutrons that are in contact leave the same place because of the action of the nuclear viscosity and the centrifugal force.

- **Remark 6:** Protons, neutrons and alpha particles leave the atomic nucleus by joint action of nuclear viscosity and the centrifugal force. The escape energy of protons  $S_p$  and the neutrons  $S_n$  are respectively the maximum allowed by the nuclear viscosity, in such a way that when an alpha particle comes out, it must have an energy functionally defined by the energies of protons and neutrons.

### 2.3. Maximum Number of Protons $Z^*$ , On The Nuclear Surface

The radius of the proton appears naturally, when all the elements of the periodic table are analyzed, under the model of two fully established physical layers, subject to the following restriction.

$$1.2 A^{1/3} \geq 1.2 N^{1/3} + r_p$$

The nuclear surface houses all the protons, which are at a maximum distance from each other, filling the following inequality.

$$4\pi (1.2 A^{1/3})^2 \geq Z\pi (r_p)^2$$

When the previous inequalities are transformed into equalities, we obtain the maximum number of protons on the nuclear surface and the average radius of the proton. The study of the whole periodic table shows that

$$3.4 < \frac{Z^*}{Z} < 4.5 \text{ for } Z \geq 20.$$

- **Proposition 7:** Minimum energy implies maximum entropy in the probability of spins.
- **Proof:** Be  $x$  the probability associated with the spin  $s = 1/2$  and  $(1-x)$  the probability associated to the spin  $s = -1/2$  we can define

entropy as  $H = x \ln(1/x) + (1-x) \ln(1/(1-x))$  the maximum value  $\max(H) = \ln(2)$  has to be calculated for  $x = 1/2$ . Which indicates that the spins in pairs are always annulled independently for protons and neutrons.

### 2.4. Connection Between The Holographic Principle and The Navier Stokes Equations

- **Theorem 8:** The atomic nucleus fulfils the holographic principle, for the minimal Bekenstein-Hawking entropy.

- **Proof:** The entropy of a black hole is maximum

and equal to  $S_{BH} = \frac{1}{4} \frac{c^3 k}{G \hbar} \tilde{A}$ . According to

Shannon, the maximum entropy of the atomic nucleus is given by  $S = (N + Z) \ln(2)$  to harmonize these two formulations we can represent the area of the event horizon  $\tilde{A}$  by the number of neutrons plus the number of protons

$$S_{BH} = \frac{1}{4} \frac{c^3 k}{G \hbar} (N + Z)$$

$(N + Z)$ , getting:

Using the spherical trajectory theorem given by the incompressibility of a fluid, we know that the information in the external layer of protons represents the total information of the atomic nucleus, with which the entropy of the atomic nucleus represents the minimum entropy under the formulation of Bekenstein-Hawking and has the form:

$$S_{BH} = \frac{1}{4} \frac{c^3 k}{G \hbar} (Z) \quad [1,2,3] \text{ this give us.}$$

$$\frac{1}{4} \frac{c^3 k}{G \hbar} Z < \frac{1}{4} \frac{c^3 k}{G \hbar} (N + Z)$$

### 2.5. Navier-Stokes Force

The atomic nucleus is an incompressible fluid, justified by the formula of the nuclear radius,  $R = 1.2 A^{1/3}$ , where it is evident that the volume of the atomic nucleus changes linearly with  $A = Z + N$ , giving a density constant. All incompressible fluid and especially the atomic nucleus comply with the Navier Stokes equations (1,2,3,4,5,6). We present a rigorous demonstration on the incomprehensibility of the atomic nucleus, which allows to write explicitly the form of the nuclear

force  $F_N = -\frac{g \mu^2}{8\pi} (A-1) P(1-P) \nabla r$ , which

facilitates the understanding of alpha decay [12,13].

For our demonstrations, we will use strictly the scheme presented by Fefferman in <http://www.claymath.org/millennium-problems>, where six demonstrations are required to accept as valid a solution to the Navier-Stokes 3D equation. [14, 16, 17].

The velocity defined as  $\mathbf{u} = -2\nu \frac{\nabla p}{\rho}$  with a radius noted as  $r = (x^2 + y^2 + z^2)^{1/2}$ , where  $P(x, y, z, t)$  is the logistic probability function  $P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu r}}$ , and the expected value  $E(r | r \geq 0) < C$  exist. The term  $P$  is defined in  $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ , where constants  $k > 0$ ,  $\mu > 0$  and  $P(x, y, z, t)$  is the general solution of the Navier-Stokes 3D equation, which has to satisfy the conditions (1) and (2), allowing us to analyze the dynamics of an incompressible Fluid.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{\nabla p}{\rho_0} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (1)$$

With,  $\mathbf{u} \in \mathbb{R}^3$  an known velocity vector,  $\rho_0$  constant density of Fluid,  $\eta$  dynamic viscosity,  $\nu$  cinematic viscosity, and pressure  $p = p_0 P$  in  $((x, y, z) \in \mathbb{R}^3, t \geq 0)$ .

Where velocity and pressure are depending of  $r$  and  $t$ . We will write the condition of incompressibility.

$$\nabla \cdot \mathbf{u} = 0 \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (2)$$

The initial conditions of fluid movement  $\mathbf{u}^0(x, y, z)$ , are determined for  $t = 0$ . Where speed  $\mathbf{u}^0$  must be  $C^\infty$  divergence-free vector.

$$\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) \quad ((x, y, z) \in \mathbb{R}^3) \quad (3)$$

For physically reasonable solutions, we make sure  $\mathbf{u}(x, y, z, t)$  does not grow large as  $r \rightarrow \infty$ . We will restrict attention to initial conditions  $\mathbf{u}^0$  that satisfy.

$$\left| \partial_x^\alpha \mathbf{u}^0 \right| \leq C_{\alpha\kappa} (1+r)^{-\kappa} \quad \text{on } \mathbb{R}^3 \text{ for any } \alpha \text{ and } \kappa \quad (4)$$

The Clay Institute accepts a physically reasonable solution of (1), (2) and (3), only if it satisfies.

$$p, u \in C^\infty(\mathbb{R}^3 \times [0, \infty)) \quad (5)$$

and the finite energy condition (Clay Mathematics Institute, 2017; Leray, 1934).

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C \quad \text{for all } t \geq 0 \text{ (bounded energy).} \quad (6)$$

The problems of Mathematical Physics are solved by the Nature, guiding the understanding, the scope, the limitations and the complementary theories. These guidelines of this research were: the probabilistic elements of Quantum Mechanics, the De Broglie equation and the Heisenberg Uncertainty principle.

## 2.6. Definitions

- **Nuclear Cavitation:** Formation of helium nucleus (cavities) within the nuclear fluid. Cavitations occurs when the pressure at one point of the fluid is lower than the average pressure of the fluid.
- **Nuclear viscosity:** It is created by the friction between the layers of protons and the sphere of neutrons that make up the atomic nucleus.
- **Fullerene architecture:** It allows explaining how protons are placed symmetrically in the outer layer of the atomic nucleus. All the magic numbers are in accordance with the fullerene architecture demonstrated, for  $Z = 8, 20, 28, 40, 50, 70, 72, 82$ . Neutron numbers depend on the number of protons with the ratio  $N = 1.6567Z - 12.697$  with a statistical adjustment of  $(R^2) = 0.9953$ .

### Attenuation Coefficient

We will use the known attenuation formula of an incident flux  $I_0$ , for which  $I = I_0 e^{-\mu r}$ . Where,  $I_0$  initial flux and  $\mu$  attenuation coefficient of energetic molecules that enters into interaction and/or resonance with the target molecules, transmitting or capturing the maximum amount of energy [18].

▪ **Growth Coefficient**

We will use an equation analogous to concentration equation of Physical Chemistry  $C = C_0 e^{kt}$ , where  $k = \frac{p_0}{2\eta}$ , is growth coefficient,  $p_0$  is the initial pressure of our fluid,  $\eta$  the dynamic viscosity and  $C_0$  the initial concentration of energetic fluid molecules. It is evident that, in equilibrium state we can write  $\mu r = kt$ , however, the Navier-Stokes equation precisely measures the behavior of the fluids out of equilibrium, so that:  $\mu r \neq kt$ .

Fortunately, there is a single solution for out-of-equilibrium fluids, using the fixed-point theorem for implicit functions,  $\frac{1}{1+e^{kt-\mu r}} = \frac{2}{\mu r}$ , the proof is demonstrated in Theorem 1.

▪ **Dimensional Analysis**

We will define the respective dimensional units of each one of variables and physical constants that appear in the solution of the Navier-Stokes 3D equation.

Kinematic viscosity,  $\nu = \frac{\eta}{\rho_0}$ , [ $\frac{m^2}{s}$ ]

Dynamic viscosity  $\eta$ , [p a .s], where **pa** represents Pascal pressure unit.

Initial Pressure of out of equilibrium.

$$p_0, [\text{pa}]$$

Fluid density  $\rho_0$ , [ $\frac{kg}{m^3}$ ], where  $kg$  is kilogram and  $m^3$  cubic meters.

Logistic Probability function,

$$P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu r}}, \text{it is a real number}$$

$$0 \leq P \leq 1.$$

Equilibrium-condition,

$$r = \frac{k}{\mu} t = \frac{P_0}{w \rho_0 \nu \mu} t = |\mathbf{u}_e| t, \quad [\text{m}].$$

Fluid velocity in equilibrium,  $|\mathbf{u}_e|$ , [m/s].

Fluid field velocity out of equilibrium,  $\mathbf{u} = -2\nu\mu(1-P)\nabla r$ . [m/s].

Position,  $r = (x^2 + y^2 + z^2)^{1/2}$ . [m]

Attenuation coefficient,  $\mu$ , [1/m].

Growth coefficient,  $k = \frac{P_0}{2\rho_0\nu}$ , [1/s]

Concentration  $C = C_0 \frac{1-P}{P}$ .

▪ **Efficient Frontier**

Spherical surface of an implicit function  $f(t, r) = 0$  of time  $t$  and radius  $r = (x^2 + y^2 + z^2)^{1/2}$ , which represents the solution set of the Navier-Stokes 3D equation. Every moving particle or fluid has energy measured with some standard deviation  $\langle (E - \bar{E})^2 \rangle^{1/2}$ . By the Heisenberg principle of uncertainty we know that there exists an unavoidable uncertainty in time  $\langle (t - \bar{t})^2 \rangle^{1/2}$  given by  $\langle (E - \bar{E})^2 \rangle^{1/2} \langle (t - \bar{t})^2 \rangle^{1/2} \geq \frac{h}{2\pi}$ . Where  $h$  is Planks Constant.

➤ **Theorem 9:** The velocity of the fluid is given by  $\mathbf{u} = -2\nu \frac{\nabla P}{P}$ , where  $P(x, y, z, t)$  is the logistic probability function

$$P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu r}}, \text{ and } p \text{ pressure}$$

such that  $p = p_0 P$ , both defined on

$((x, y, z) \in \mathbb{R}^3, t \geq 0)$ . The function  $P$  is the general solution of the Navier Stokes equations, which satisfies conditions (1) and (2).

➤ **Proof:** To verify condition (2),  $\nabla \cdot \mathbf{u} = 0$ , we must calculate the gradients and laplacians of the radius.

$$\nabla r = \left( \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right), \quad \text{and}$$

$$\nabla^2 r = \nabla \cdot \nabla r = \frac{(y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2)}{(x^2 + y^2 + z^2)^{3/2}} = \frac{2}{r}$$

$$\nabla \cdot \mathbf{u} = -2\nu \nabla \cdot \frac{\nabla P}{P} = -2\nu \mu \nabla \cdot ((1-P)\nabla r) \quad (7)$$

Replacing the respective values for the terms:  $\nabla^2 r$  and  $|\nabla r|^2$  in the equation(7).

$$\begin{aligned} \nabla \cdot \mathbf{u} &= -2\nu\mu\nabla((1-P)\nabla r) \\ &= -2\nu\mu\nabla((1-P)\nabla r) \\ &= -2\nu\mu[-\mu(P-P^2)|\nabla r|^2 + (1-P)\nabla^2 r] \end{aligned} \quad (8)$$

Where the gradient modulus of  $\nabla P = \mu(P-P^2)\nabla r$ , has the form  $|\nabla P|^2 = \mu^2(P-P^2)^2|\nabla r|^2 = \mu^2(P-P^2)^2$ .

$$\nabla \cdot \mathbf{u} = -2\nu\mu(1-P) \left[ -\mu P + \frac{2}{r} \right] = 0 \quad (9)$$

Simplifying for  $(1-P) \neq 0$ , we obtain the main result of this paper, which represents a fixed point of an implicit function  $f(t, r)$  where  $f(t, r) = P - \frac{2}{\mu r} = 0$ .

In Nuclear Physics,  $r_0 < r < 1.2 A^{1/3}$ .

$$P = \frac{1}{1+e^{kt-\mu(x^2+y^2+z^2)^2}} = \frac{2}{\mu(x^2+y^2+z^2)^2} \quad ((x, y, z) \in \mathbb{R}^3, t \geq 0) \quad (10)$$

Equation (10) has a solution according to the fixed-point theorem of an implicit function, and it is a solution to the Navier Stokes stationary equations, which are summarized in:  $\nabla^2 P = \frac{2}{\mu} \nabla^2 \left(\frac{1}{r}\right) = 0$ . Furthermore, it is the typical solution of the Laplace equation for the pressure of the fluid  $\nabla^2 p = p_0 \nabla^2 P = 0$ . Kerson Huang (1987).

To this point, we need to verify that equation (10) is also a solution of requirement (1),  $\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \nabla^2 \mathbf{u} - \frac{\nabla p}{\rho_0}$ . We will do the equivalence  $\mathbf{u} = \nabla \theta$ , after we replace in equation (1). Taking into account that  $\theta = -2\nu \ln(P)$ , and that  $\nabla \theta$  is irrotational,  $\nabla \times \nabla \theta = 0$ , we have:  $(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \theta \cdot \nabla) \nabla \theta = \frac{1}{2} \nabla(\nabla \theta \cdot \nabla \theta) - \nabla \theta \times (\nabla \times \nabla \theta) = \frac{1}{2} \nabla(\nabla \theta \cdot \nabla \theta)$ , and  $\nabla^2 \mathbf{u} = \nabla(\nabla \cdot \mathbf{u}) - \nabla \times (\nabla \times \mathbf{u}) = \nabla(\nabla \cdot \nabla \theta) - \nabla \times (\nabla \times \nabla \theta) = \nabla(\nabla^2 \theta)$ . Simplifying terms in order to replace these results in equation (1) we obtain

$$\begin{aligned} (\mathbf{u} \cdot \nabla) \mathbf{u} &= \frac{1}{2} \nabla(\nabla \theta \cdot \nabla \theta) = 2\nu^2 \nabla \left( \frac{|\nabla P|^2}{P^2} \right) \\ \nabla^2 \mathbf{u} &= \nabla(\nabla \cdot \mathbf{u}) = \nabla(\nabla^2 \theta) = 0 \\ &= -2\nu \nabla \left( \frac{|\nabla P|^2}{P^2} - \left( \frac{\nabla^2 P}{P} \right) \right) = 0 \end{aligned}$$

The explicit form of velocity is  $\mathbf{u} = -2\nu\nabla \ln(P)$ . Next, we need the partial derivative  $\frac{\partial \mathbf{u}}{\partial t}$

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} &= -2\nu k P(1-P) \nabla r, \\ -\frac{\nabla p}{\rho_0} &= -\frac{\mu P_0}{\rho_0} P(1-P) \nabla r. \end{aligned}$$

After replacing the last four results  $(\mathbf{u} \cdot \nabla) \mathbf{u}$ ,  $\nabla^2 \mathbf{u}$ ,  $\frac{\partial \mathbf{u}}{\partial t}$  and  $-\frac{\nabla p}{\rho_0}$  in equation (1) we obtain (11).

$$-2\nu k P(1-P) \nabla r = 2\nu^2 \nabla \left( \frac{|\nabla P|^2}{P^2} \right) - \frac{\mu P_0}{\rho_0} P(1-P) \nabla r \quad (11)$$

The equation (11) is equivalent to equation (1). After obtaining the term  $\frac{|\nabla P|^2}{P^2}$  from the incompressibility equation  $\nabla(\nabla^2 \theta) = -2\nu \nabla \left( -\frac{|\nabla P|^2}{P^2} + \frac{\nabla^2 P}{P} \right) = 0$  and replacing in equation (11).

$$-2\nu k P(1-P) \nabla r = 2\nu^2 \nabla \left( \frac{\nabla^2 P}{P} \right) - \frac{\mu P_0}{\rho_0} P(1-P) \nabla r \quad (12)$$

Equation (10) simultaneously fulfills requirements (1) expressed by equation (12) and requirement (2) expressed by equation (7), for a constant  $k = \frac{P_0}{2\rho_0\nu} = \frac{P_0}{2\eta}$ . Moreover, according to equation (10), the probability  $p = \frac{2}{\mu r}$  which allows the Laplace equation to be satisfied:

$\nabla^2 P = \frac{2}{\mu} \nabla^2 \left( \frac{1}{r} \right) = 0$ . In other words, the Navier-Stokes 3D equation system is solved.

▪ **Implicit Function**

An implicit function defined as (10),  $f(t, r) = \frac{1}{1 + e^{kt - \mu r}} - \frac{2}{\mu r} = 0$ , has a fixed point

$(t, r)$  of  $R = \{(t, r) \mid 0 < a \leq t \leq b, 0 < r < +\infty\}$ , where  $m$  and  $M$  are constants, such as:  $m \leq M$ . Knowing that the partial derivative exists.  $\partial_r f(t, r) = vP(1 - P) + \frac{2}{\mu r^2}$  we can assume that:

$0 < m \leq \partial_r f(t, r) \leq M$ . If, in addition, for each continuous function  $\varphi$  in  $[a, b]$  the composite function  $g(t) = f(t, \varphi(t))$  is continuous in  $[a, b]$ , then there is one and only one function  $r = \varphi(t)$  Continuous in  $[a, b]$ , such that  $f[t, \varphi(t)] = 0$  for all  $t$  in  $[a, b]$ .

➤ **Theorem 10:** An implicit function defined as (10)  $f(t, r) = \frac{1}{1 + e^{kt - \mu r}} - \frac{2}{\mu r} = 0$  has a fixed point  $(t, r)$  of  $R = \{(t, r) \mid 0 < a \leq t \leq b, 0 < r < +\infty\}$ . In this way, the requirements (1) and (2) are fulfilled.

➤ **Proof:** Let  $C$  be the linear space of continuous functions in  $[a, b]$ , and define an operator  $T : C \rightarrow C$  by the

➤ equation:  $T \varphi(t) = \varphi(t) - \frac{1}{M} f[t, \varphi(t)]$

Then we prove that  $T$  is a contraction operator, so it has a unique fixed point  $r = \varphi(t)$  in  $C$ . Let us construct the following distance.

$$T \varphi(t) - T \psi(t) = \varphi(t) - \psi(t) - \frac{f[t, \varphi(t)] - f[t, \psi(t)]}{M}$$

Using the mean value theorem for derivation, we have

$$f[t, \varphi(t)] - f[t, \psi(t)] = \partial_\phi f(t, z(t))[\varphi(t) - \psi(t)].$$

Where  $\phi(t)$  is situated between  $\varphi(t)$  and  $\psi(t)$ . Therefore, the distance equation can be written as:

$$T \varphi(t) - T \psi(t) = [\varphi(t) - \psi(t)] \left[ 1 - \frac{\partial_\phi f(t, z(t))}{M} \right]$$

Using the hypothesis  $0 < m \leq \partial_r f(t, r) \leq M$  we arrive at the following result:

$$0 \leq 1 - \frac{\partial_\phi f(t, \phi(t))}{M} \leq 1 - \frac{m}{M}$$

With which we can write the following inequality.

$$|T \varphi(t) - T \psi(t)| = |\varphi(t) - \psi(t)| \left( 1 - \frac{m}{M} \right) \leq \alpha \|\varphi - \psi\| \tag{13}$$

Where  $\alpha = \left( 1 - \frac{m}{M} \right)$ . Since  $0 < m \leq M$ , we have  $0 \leq \alpha < 1$ . The above inequality is valid for all  $t$  of  $[a, b]$ . Where  $T$  is a contraction operator and the proof is complete, since for every contraction operator  $T : C \rightarrow C$  there exists one and only one continuous function  $\varphi$  in  $C$ , such that  $T(\varphi) = \varphi$ . Using equation (10), which represents the fundamental solution of the Navier-Stokes 3D equation, we verify equation (2), which represents the second of the six requirements of an acceptable solution.

➤ **Proposition 11** Requirement (3). The initial velocity can be obtained from:  $\mathbf{u}(x, y, z, 0) = -2v \frac{\nabla P}{P}$  where each of the components  $u_x, u_y$  and  $u_z$  are infinitely derivable.

$$\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) = -2v \mu (1 - P_0) \begin{pmatrix} x & y & z \\ - & - & - \\ r & r & r \end{pmatrix}, \quad ((x, y, z) \in \mathbb{R}^3)$$

$$P_0 = \frac{1}{1 + e^{-\mu r_0}}$$

(14)

➤ **Proof.** Taking the partial derivatives of  $\partial_x^n \left( \frac{x}{r} \right)$ ,  $\partial_y^n \left( \frac{y}{r} \right)$  and  $\partial_z^n \left( \frac{z}{r} \right)$

$$\begin{aligned} \partial_x^n \left(\frac{x}{r}\right) &= n \partial_x^{n-1} \left(\frac{1}{r}\right) + x \partial_x^n \left(\frac{1}{r}\right) \\ \partial_y^n \left(\frac{x}{r}\right) &= n \partial_y^{n-1} \left(\frac{1}{r}\right) + y \partial_y^n \left(\frac{1}{r}\right) \\ \partial_z^n \left(\frac{x}{r}\right) &= n \partial_z^{n-1} \left(\frac{1}{r}\right) + z \partial_z^n \left(\frac{1}{r}\right) \end{aligned} \tag{15}$$

Recalling the derivatives of special functions (Legendre), it is verified that there exists the derivative  $C^\infty$ .

$$\begin{aligned} \partial_x^n \left(\frac{1}{r}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{x}{(x^2 + y^2 + z^2)^{1/2}}\right) \\ \partial_y^n \left(\frac{1}{r}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{y}{(x^2 + y^2 + z^2)^{1/2}}\right) \\ \partial_z^n \left(\frac{1}{r}\right) &= (-1)^n n! (x^2 + y^2 + z^2)^{-\frac{(n+1)}{2}} P_n \left(\frac{z}{(x^2 + y^2 + z^2)^{1/2}}\right) \end{aligned} \tag{16}$$

Physically, this solution is valid for the initial velocity, indicated by Eq. (4), where the components of the initial velocity are infinitely differentiable, and make it possible to guarantee that the velocity of the fluid is zero when  $r \rightarrow \infty$  [14].

➤ **Proposition 12** Requirement (4). Using the initial velocity of a moving fluid given by  $\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) = -2v\mu(1 - P_0)\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$  it is evident that

$$\left| \partial_x^\alpha \mathbf{u}^0 \right| \leq C_{\alpha K} (1+r)^{-K} \quad \text{on } \mathbb{R}^3 \text{ for any } \alpha \text{ and } K$$

➤ **Proof:** Using the initial velocity of a moving fluid given by  $\mathbf{u}^0(x, y, z) = -2v\mu(1 - P_0)\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$ , we can find each of the components  $\partial_x^\alpha u_x^0$ ,  $\partial_y^\alpha u_y^0$  and  $\partial_z^\alpha u_z^0$

$$\left(\partial_x^\alpha \frac{x}{r}\right)^2 = \left(\alpha \partial_x^{\alpha-1} \left(\frac{1}{r}\right) + x \partial_x^\alpha \left(\frac{1}{r}\right)\right) \left(\alpha \partial_x^{\alpha-1} \left(\frac{1}{r}\right) + x \partial_x^\alpha \left(\frac{1}{r}\right)\right)$$

For the three components  $x, y, z$  the results of the partial derivatives are as follows:

$$\begin{aligned} \left(\partial_x^\alpha \frac{x}{r}\right)^2 &= \alpha^2 \left(\partial_x^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha x \partial_x^{\alpha-1} \frac{1}{r} \partial_x^\alpha \frac{1}{r} + x^2 \left(\partial_x^\alpha \frac{1}{r}\right)^2 \\ \left(\partial_y^\alpha \frac{y}{r}\right)^2 &= \alpha^2 \left(\partial_y^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha y \partial_y^{\alpha-1} \frac{1}{r} \partial_y^\alpha \frac{1}{r} + y^2 \left(\partial_y^\alpha \frac{1}{r}\right)^2 \\ \left(\partial_z^\alpha \frac{z}{r}\right)^2 &= \alpha^2 \left(\partial_z^{\alpha-1} \frac{1}{r}\right)^2 + 2\alpha z \partial_z^{\alpha-1} \frac{1}{r} \partial_z^\alpha \frac{1}{r} + z^2 \left(\partial_z^\alpha \frac{1}{r}\right)^2 \end{aligned} \tag{17}$$

Replacing equation (17) with the explanatory form of the Legendre polynomials, for the following terms  $\partial_x^{\alpha-1} \frac{1}{r}$  and  $\partial_x^\alpha \frac{1}{r}$ .

$$\begin{aligned} \partial_x^{\alpha-1} \frac{1}{r} &= (-1)^{\alpha-1} \alpha! (x^2 + y^2 + z^2)^{-\frac{(\alpha+1)}{2}} P_{\alpha-1} \left(\frac{x}{(x^2 + y^2 + z^2)^{1/2}}\right) \\ \partial_x^\alpha \frac{1}{r} &= (-1)^\alpha (\alpha-1)! (x^2 + y^2 + z^2)^{-\frac{\alpha}{2}} P_{\alpha-1} \left(\frac{x}{(x^2 + y^2 + z^2)^{1/2}}\right) \end{aligned} \tag{18}$$

Also, knowing that for each  $\alpha \geq 0$ , the maximum value of  $P_\alpha(1) = 1$ . We can write the following inequality

$$\begin{aligned} x^2 \left(\partial_x^\alpha \left(\frac{1}{r}\right)\right)^2 &\leq x^2 (\alpha!)^2 r^{-2(\alpha+1)} \\ 2\alpha x \partial_x^{\alpha-1} \frac{1}{r} \partial_x^\alpha \frac{1}{r} &\leq 2\alpha x (\alpha!) (\alpha-1)! (-1)^{2\alpha-1} r^{-2\alpha-1} \\ \alpha^2 \left(\partial_x^{\alpha-1} \left(\frac{1}{r}\right)\right)^2 &\leq \alpha^2 ((\alpha-1)!)^2 r^{-2\alpha} \end{aligned}$$

Grouping terms for  $\left(\partial_x^\alpha \frac{x}{r}\right)^2, \left(\partial_y^\alpha \frac{y}{r}\right)^2$  and  $\left(\partial_z^\alpha \frac{z}{r}\right)^2$  we have the next expressions.

$$\begin{aligned} \left(\partial_x^\alpha \frac{x}{r}\right)^2 &\leq r^{-2\alpha} \left[ \frac{x^2(\alpha!)^2}{r^2} - \frac{2x(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] \\ \left(\partial_y^\alpha \frac{y}{r}\right)^2 &\leq r^{-2\alpha} \left[ \frac{y^2(\alpha!)^2}{r^2} - \frac{2y(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] \\ \left(\partial_z^\alpha \frac{z}{r}\right)^2 &\leq r^{-2\alpha} \left[ \frac{z^2(\alpha!)^2}{r^2} - \frac{2z(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2 \right] \end{aligned} \quad (20)$$

The module of  $\left|\partial_x^\alpha \mathbf{u}^0\right|$  is given by

$$\left|\partial_x^\alpha \mathbf{u}^0\right| = \left( \left(\partial_x^\alpha \frac{x}{r}\right)^2 + \left(\partial_y^\alpha \frac{y}{r}\right)^2 + \left(\partial_z^\alpha \frac{z}{r}\right)^2 \right)^{1/2}$$

Simplifying and placing the terms of equation (20) we have

$$\left|\partial_x^\alpha \mathbf{u}^0\right| \leq r^{-2\alpha} \left[ 3(\alpha!)^2 + \alpha^2((\alpha-1)!)^2 - \frac{2(x+y+z)(\alpha!)^2}{r} \right]$$

Taking into consideration that  $\left|\frac{x}{r}\right| \leq 1$ ,  $\left|\frac{y}{r}\right| \leq 1$ ,  $\left|\frac{z}{r}\right| \leq 1$  the last term  $\left|\partial_x^\alpha \mathbf{u}^0\right|$  can be easily written that.

$$\left|\partial_x^\alpha \mathbf{u}^0\right| \leq \frac{2(\alpha!)^2}{r^{2\alpha}} \left[ 2 + \left|\frac{x}{r}\right| + \left|\frac{y}{r}\right| + \left|\frac{z}{r}\right| \right]$$

$$\left|\partial_x^\alpha \mathbf{u}^0\right| \leq \frac{10(\alpha!)^2}{r^{2\alpha}}$$

It is verified that there exists  $C_\alpha = 10(\alpha!)^2$  such that if  $r \rightarrow 0$ , then  $\left|\partial_x^\alpha \mathbf{u}^0\right| \rightarrow 0$ . Thus, we proved requirement (4). According to Mathematics, and giving an integral physical structure to the study, we need to prove that there are the spatial and temporal derivatives of the velocity and pressure components, satisfying the requirement (5).

➤ **Proposition 13:** Requirement (5). The velocity can be obtained from:  $\mathbf{u}(x, y, z, t) = -2v \frac{\nabla P}{P}$  and each of the components  $u_x$ ,  $u_y$  and  $u_z$  are infinitely derivable.

$$u(x, y, z, t) = -2v^2 \left( \frac{x}{r^2}, \frac{y}{r^2}, \frac{z}{r^2} \right),$$

$$((x, y, z) \in \mathbb{R}^3)$$

$$P(x, y, z, t) = \frac{1}{1 + e^{\frac{P_0}{2n}t - \mu r}} = \frac{2}{\mu r} \quad (21)$$

➤ **Proof:** Taking partial derivatives for  $\partial_x^n \left(\frac{x}{r^2}\right)$ ,  $\partial_y^n \left(\frac{x}{r^2}\right)$  and  $\partial_z^n \left(\frac{x}{r^2}\right)$ .

$$\partial_x^n \left(\frac{x}{r^2}\right) = n \partial_x^{n-1} \left(\frac{1}{r^2}\right) + x \partial_x^n \left(\frac{1}{r^2}\right)$$

$$\partial_y^n \left(\frac{x}{r^2}\right) = n \partial_y^{n-1} \left(\frac{1}{r^2}\right) + y \partial_y^n \left(\frac{1}{r^2}\right)$$

$$\partial_z^n \left(\frac{x}{r^2}\right) = n \partial_z^{n-1} \left(\frac{1}{r^2}\right) + z \partial_z^n \left(\frac{1}{r^2}\right)$$

Recalling the derivatives of special functions, it is verified that the derivative  $C^\infty$  exists. These derivatives appear as a function of the Legendre polynomials  $P_n(\cdot)$ .

$$\partial_x^n \left(\frac{1}{r^2}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{x}{(x^2 + y^2 + z^2)}\right)$$

$$\partial_y^n \left(\frac{1}{r^2}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{y}{(x^2 + y^2 + z^2)}\right)$$

$$\partial_z^n \left(\frac{1}{r^2}\right) = (-1)^n n! (x^2 + y^2 + z^2)^{-(n+1)} P_n \left(\frac{z}{(x^2 + y^2 + z^2)}\right) \quad (23)$$

There are the spatial derivatives  $n$  and the time derivative which is similar to equations (25).

➤ **Proposition 14:** Requirement (5). The pressure is totally defined by the equivalence  $p(x, y, z, t) = p_0 P(x, y, z, t)$  and is infinitely differentiable in each of its components.

$$p(x, y, z, t) = p_0 P(x, y, z, t), \quad ((x, y, z) \in \square^3) \quad (24)$$

➤ **Proof:** Taking partial derivatives for and  $\partial_x^n \left(\frac{1}{r}\right)$ ,  $\partial_y^n \left(\frac{1}{r}\right)$  and  $\partial_z^n \left(\frac{1}{r}\right)$ , recalling the derivatives of special functions of equation (16), it is shown that the derivative  $C^\infty$ . We only have to find the time derivative  $\partial_t^n (p_0 P) = p_0 \partial_t^n (P)$ . Using equation (21) for  $P$ , we have.



$$\begin{aligned}
 \partial_t P &= (-k)P(1-P) \\
 \partial_t^2 P &= (-k)^2(1-2P)P(1-P) \\
 \partial_t^3 P &= (-k)^3(1-6P+6P^2)P(1-P) \\
 \partial_t^4 P &= (-k)^4(1-14P+36P^2-24P^3)P(1-P) \\
 \partial_t^5 P &= (-k)^5(1-30P+150P^2-240P^3+120P^4)P(1-P) \\
 \partial_t^n P &= \partial_t(\partial_t^{n-1}(P))
 \end{aligned}
 \tag{25}$$

It is always possible to find the derivative  $\partial_t^n P$  as a function of the previous derivative, since the resulting polynomial of each derivative  $n-1$  is of degree  $n$ .

**Proposition 15:** Requirement (6). The energy must be limited in a defined volume and fundamentally it must converge at any time, such that  $t \geq 0$ .

$$\int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \leq C \text{ for all } t \geq 0$$

(bounded energy).

➤ **Proof:** We will use the explicit form of velocity given in equation (21)  $\mathbf{u}(x, y, z, t) = 2v\mu(1-P)\nabla r$ , to obtain the vector module:  $|\mathbf{u}|^2 = 4v^2\mu^2(1-P)^2$ . Rewriting equation (21), and applying a change of variable in:  $dx dy dz = 4\pi r^2 dr$ .

$$\begin{aligned}
 \int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz \\
 = 16\pi v^2 \mu^2 \int_{r_0}^{\infty} r^2 (1-P)^2 dr
 \end{aligned}
 \tag{26}$$

Making another change of variable  $dP = \mu P(1-P)dr$ . Using (10), replacing

$$r^2 = \left( \frac{2}{\mu P} \right)^2 \text{ we have}$$

$$\begin{aligned}
 \int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz &= 16\pi v^2 \mu^2 \int_{p_0}^{P_0} \left( \frac{2}{\mu P} \right)^2 (1-P)^2 \frac{dP}{\mu P(1-P)} \\
 &= \frac{64\pi v^2}{\mu} \int_{p_0}^{P_0} \frac{1-P}{P^3} dP
 \end{aligned}
 \tag{27}$$

Where radius  $r \rightarrow \infty$ , when  $t \geq 0$ , we have

$$\lim_{r \rightarrow \infty} P = \lim_{r \rightarrow \infty} \frac{1}{1 + \frac{\exp(kt)}{\exp(\mu r)}} = P_{\infty} = 1.$$

Moreover, physically if  $r \rightarrow r_0 \approx 0$  then  $t \rightarrow 0$  we

$$\text{have } \lim_{r \rightarrow 0} P = \lim_{r \rightarrow 0} \frac{1}{1 + \frac{\exp(kt)}{\exp(\mu r)}} = P_0 = \frac{1}{2}.$$

Here, a probability  $\frac{1}{2}$  represents maximum entropy.

$$\begin{aligned}
 \int_{\mathbb{R}^3} |\mathbf{u}(x, y, z, t)|^2 dx dy dz &= \frac{64\pi v^2}{\mu} \int_{1/2}^1 \frac{1-P}{P^3} dP \\
 &= \frac{64\pi v^2}{\mu} \left[ \frac{2P-1}{2P^2} \right]_{1/2}^1
 \end{aligned}$$

$$\int_{\mathbb{R}^3} |\mathbf{u}|^2 dx dy dz \leq \frac{32\pi v^2}{\mu} \text{ for all } t \geq 0 \tag{28}$$

In this way the value of the constant  $C$  is  $C = \frac{32\pi v^2}{\mu}$ .

Verifying the proposition (6) completely. In general, equation (10) can be written

$$f(t, r + r_0) = \frac{1}{1 + e^{-\mu(r+r_0)}} - \frac{2}{\mu\mu(r+r_0)} = 0 \text{ and in this}$$

way discontinuities are avoided when  $r \rightarrow 0$ , but this problem does not occur since in the atomic nucleus  $r_0 < r < 1.2 A^{1/3}$  is satisfied.

➤ **Lemma 16:** The irrigational field represented by the logistic probability function  $P(x, y, z, t)$  associated with the velocity  $\mathbf{u} = -2v \frac{\nabla P}{P}$ , can produce vortices, due to the stochastic behavior of the physical variables  $p_0, \eta, \mu$ . These stochastic variations are in orders lower than the minimum experimental value.

➤ **Proof:** The implicit function representing the solution of the Navier-Stokes3D equation,

$$\frac{1}{1 + e^{-\mu(r - \frac{k}{\mu}t)}} = \frac{2}{\mu r}$$

depends on the values of initial pressure  $p_0$ , viscosity  $v$  and attenuation coefficient  $\mu$ . Due to Heisenberg uncertainty principle, these parameters have a variation when we measure and use them, as is the case of the estimate of  $\xi = r - \phi(t, k, \mu)$ . The function  $\phi(t, k, \mu) = \frac{k}{\mu}t$  expressly incorporates these results, when  $-\infty < \xi < +\infty$ . The physical and mathematical realities are mutually conditioned

and allow for these surprising results. For a definite  $t$  there exist infinities  $(x, y, z)$  that hold the relationship  $r = (x^2 + y^2 + z^2)^{1/2}$ . Moreover, for a definite  $r$  there is infinity's  $t$  that respects the fixed-point theorem and creates spherical trajectories. When the physical variables  $k, \mu$  vary, even at levels of 1/100 or 1/1000, they remain below the minimum variation of the experimental value. We could try to avoid the existence of trajectories on the spherical surface, for which we must assume that the fluid is at rest or it is stationary, which contradicts the Navier-Stokes 3D equation, where all fluid is in accelerated motion  $\frac{\partial \mathbf{u}}{\partial t} \neq 0$ . In short, if there are trajectories in the sphere as long as it is probabilistically possible, this is reduced to showing that the expected value of the radius  $E[r | r \geq 0]$  exists and it is finite. Derivation of  $E(r | r \geq 0)$ .

The logistic density function for  $\xi$  when  $E(\xi) = 0$  and  $\text{Var}(\xi) = \sigma^2$  is defined by  $h(\xi) = \frac{\mu \exp(-\xi)}{[1 + \exp(-\xi)]^2}$  where  $\frac{1}{\mu} = \sigma \sqrt{3} / \pi$  is a scale parameter. Given that  $r = \phi(t, p_0, \eta, \mu) + \xi$  function for  $r$  is then  $f(r) = \frac{\exp[-r - \phi(\bullet)/\tau]}{\tau [1 + \exp(-(r - \phi(\bullet))/\tau)]^2}$  to facilitate the calculations we put  $\phi(\bullet) = \phi(t, k, \mu) = \frac{k}{\mu} t$ . By definition, the truncated density for  $r$  when  $r \geq 0$  is given by  $f(r | r \geq 0) = \frac{f(r)}{P(r \geq 0)}$  for  $r \geq 0$ . Given that the cumulative distribution function for  $r$  is given by  $F(r) = \frac{1}{1 + \exp(kt - \mu r)}$ , it follows that  $P(r \geq 0) = 1 - F(0) = \frac{\exp(\phi(\bullet))}{1 + \exp(\phi(\bullet))} = \frac{1}{1 + \exp(-\phi(\bullet))}$ . The derivation of  $E(r | r \geq 0)$  then proceeds as follows:

$$E(r | r \geq 0) = \int_0^{\infty} \mu r f(r | r \geq 0) dr = \frac{1}{P(r \geq 0)} \int_0^{\infty} \mu r \frac{\exp[kt - \mu r]}{[1 + \exp[kt - \mu r]]^2} dr$$

$$E(r | r \geq 0) = \frac{1}{P(r \geq 0)^{1/2}} \frac{2}{\mu P} \frac{dP}{P(1-P)} \quad (29)$$

We replaced in equation (29)  $dP = \mu P(1-P)dr$  and  $r^2 = (\frac{2}{\mu P})^2$  of this manner we obtain

$$E(r | r \geq 0) = \frac{1}{P(r \geq 0)^{1/2}} \frac{2}{\mu P} (\mu P(1-P)) \frac{dP}{P(1-P)} = \frac{1}{P(r \geq 0)} \frac{2}{\mu} \log(2) \quad (30)$$

Where we have used the fact that

$$P(r \geq 0) = \frac{\exp(kt)}{1 + \exp(kt)}$$

$$E(r | r \geq 0) = \frac{1}{P(r \geq 0)} \frac{2}{\mu} \log(2) \leq \frac{2}{\mu} \log(2) \quad \text{for all } t \geq 0 \quad (31)$$

where the last equality follows from an application of the L'Hopital's rule  $P(r \geq 0) = \lim_{t \rightarrow \infty} \frac{\exp(kt)}{1 + \exp(kt)} = 1$ .

## 2.7. The Nuclear Force and the Navier Stokes Force are the Same

Firstly, we will use the concepts of Classic Mechanics and the formulation of the Yukawa potential,  $\Phi(r) = \frac{g}{4\pi r} (A-1) e^{-\mu r}$  to find the nuclear force exerted on each nucleon at interior of the atomic core  $\mathbf{F}_N = -\nabla \Phi(r)$ . Also, replace the terms of the potential  $e^{-\mu r} = \frac{1-P}{P}$  and  $\frac{1}{r} = \frac{\mu}{2} P$  by the respective terms already obtained in equation (10).

$$\Phi(r) = \frac{g(A-1)}{4\pi r} e^{-\mu r} = \frac{g\mu(A-1)}{8\pi} \quad (32)$$

The general form of the equation (32), is a function of  $(x, y, z, t)$ .

$$\Phi(r, t) = \frac{g(A-1)}{4\pi r} e^{kt - \mu r} = \frac{g\mu(A-1)}{8\pi} (1 - P(r, t)) \quad (33)$$

Secondly, we will obtain the Navier-Stokes force equation given by:

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + 2v^2 \nabla \left( \frac{|\nabla P|^2}{P^2} \right) = -2\mu v k P(1-P) \nabla \mathbf{r}$$

➤ **Theorem17:** The Nuclear Force and Navier Stokes Force are proportional inside the atomic nucleus  $\mathbf{F}_N = C \mathbf{F}_{NS}$ .

➤ **Proof:** Equation (34) rigorously demonstrated by theorems and propositions 1 through 8, represent the acceleration of a particle within the atomic nucleus. According to Classical Mechanics the force of Navier Stokes applied to a particle of mass  $m$ , would have the form:

$$\mathbf{F}_{NS} = m \frac{d\mathbf{u}}{dt} = -2m \mu \nu k P (1 - P) \nabla \mathbf{r} \quad (35)$$

➤ **Proof:** The nuclear force on its part would be calculated as follows  $\mathbf{F}_N = -\nabla \Phi(r)$ .

$$\mathbf{F}_N = -\nabla \Phi(r) = -\frac{g \mu}{8\pi} (A - 1) \nabla P \quad (36)$$

Replacing the term  $\nabla P = \mu (P - P^2) \nabla r$  of equation (7), we obtain

$$\mathbf{F}_N = -\nabla \Phi(r) = -\frac{g \mu^2}{8\pi} (A - 1) P (1 - P) \nabla \mathbf{r} \quad (37)$$

It is possible to write nuclear force as a function of speed.

$$\mathbf{F}_N = -\nabla \Phi(r) = -\frac{g \mu^2}{8\pi} (A - 1) P (1 - P) \nabla \mathbf{r} .$$

Finally, we can show that the nuclear force and force of Navier-Stokes differ at most in a constant  $C$ . Equating (35) and (37), we find the value  $g$  as a function of the parameters nuclear viscosity  $\nu$ , attenuation  $\mu$  and growth coefficient of the nuclear reaction  $k$ , nucleon mass  $m$  and  $C \neq 1$ .

$$g = \frac{16m\pi\nu k}{\mu(A-1)} C \quad (38)$$

### 3. Results

#### 3.1. Theory and Experiment A Dynamic Relationship

Theory or experiment? This question was until the last century, today is two paths of knowledge of physical reality, which complement each other, interact and are intimately related. The one serves the

other and vice versa. Successful theoretical models have been validated experimentally. And they have a strict mathematical representation. On the other hand, the experiments with a high degree of precision do not need theoretical verification but rather foundation and explanation with predictive character.

#### 3.2. Theory and Experiment Two Complementary Elements

When the theoretical formalization of a physical phenomenon, such as the total absorption of low energy x-rays in the K-edge[11] is well defined and can be reproduced by the simulation path with a minimum error, it is said that simulation is equivalent to experimentation. This is the case of the GEANT4 software [16]. When the experimentation is carried out with a high degree of precision and control of variables that could alter the measurement, it is said that there is a value of one variable with maximum confidence and minimum error. And the laboratory that presents the best standards becomes a world reference laboratory; this is the case of NIST. The maximum excess mass and the maximum excess effective section are correlated with dark matter. Analyzing the figures 1, 2, 4 it is concluded that the elements chrome and thallium detect dark matter, for the following reasons:

- It is no coincidence that the NIST and GEANT4 data differ only for two elements of the periodic table, while for the rest of the elements the match is perfect,  $R^2 \rightarrow 1$ .
- It is verified experimentally that only the maximum value of excess mass interacts with dark matter at the nuclear surface. Therefore only resonances of interaction between baryonic matter-dark material for chromium and thulium are present.
- For the study of dark matter and its interaction with the excess of nuclear mass, it is necessary to start the Femtoscope that is obtained by the interaction of low energy x-rays with the atomic nucleus. The Femtoscope involves the discernment and differentiation of each atom. In addition, the quantification of the nucleons present in the atomic nucleus, the measurement of the proton and neutron radii, the nuclear stability and the variation of the cross section by the presence of dark matter. This equipment increases the level of accuracy in atomic and nuclear measurements of nanometers characteristic of the atom to the order of femtometers characteristic of nucleons. Theorem 1.2, Lemma 3.

### 3.3. Dark Matter, Invisible or Hidden is Synonyms

The name dark matter refers to matter that does not interact directly with electromagnetic radiation. It is also called invisible matter because we cannot see it. We will call it hidden matter because it hides between the nucleons of the nuclear surface. It was proved theoretically and experimentally that the strong nuclear force maintains the quasi-spherical shape of the incompressible atomic nucleus, while the Navier-Stokes force controls both the circular trajectories of the nucleon layers in the atomic nucleus and the interaction with dark matter (hidden, invisible). The strategic variable of the movement of the nuclear layers is the nuclear viscosity, which coincides with previously obtained values. Categorically in the scientific community of dark matter it is accepted that it obeys only the weak nuclear force and the gravitational force. In this sense, it is necessary to deepen the knowledge of the weak interaction in the vicinity of the nuclear surface and inside the atomic nucleus. This objective is fulfilled by the solution of the Navier Stokes equations, which is one of the three cornerstones of this research. Excess ratio of resonance cross section

$\left( \left( \frac{\sigma_1}{\sigma_2} \right)_{GEANT4} - \left( \frac{\sigma_1}{\sigma_2} \right)_{NIST} \right)$ . The difference of values of the relationships of effective sections between the GEANT4 simulation and the NIST results. The GEANT4 simulation works with high precision and takes into consideration the physical processes of baryonic matter. The NIST experiment delivers results of effective sections of baryonic matter and dark matter. Excess Energy  $(E_{GEANT4} - E_{NIST})$ . The difference in energy values of the x-ray used in the GEANT4 simulation minus the energy value of the x-ray used in the NIST experiment. This difference appears for xenon. Excess mass  $(Zm_p + Nm_n - M(Z; N))$ . It is the theoretical mass difference of an atom  $(Zm_p + Nm_n)$  with respect to the experimental mass measurement of that same atom  $M(Z; N)$ . The most notorious values of the nuclear mass deficit occur for the atoms Cr, Mn, Fe, Co, Ni. However, the maximum value occurs only for Fe, which has been used as a reference for nuclear stability. The mass deficit is correlated with the excess effective section for the Cr and the Tm. The question is what implications do this excess mass have? A viable answer has to do with the presence of dark matter.

The equations that allow the calculation of the excess of cross section are the following:

$$\frac{8000\pi r \lambda}{(\sigma_2 - \sigma_1)} = R_\infty \left( \frac{\sigma_1}{\sigma_2} \right)^{2.5031}$$

Last equation with a  $R^2 = 0.9935$  was demonstrated and constructed using the elements of the solution of the Navier-Stokes equations.

$$\left( \frac{\sigma_1}{\sigma_2} \right) = 0.0021Z + 0.00696$$

This equation was obtained with a  $R^2 = 0.9939$ , and indicates that the ratio of the effective sections fully explain each element of the periodic table.

$$E = 2 * 10^{-5} Z^2 - 0.0003Z + 0.004$$

This equation obtained for a  $R^2 = 0.9996$ , complements the system of equations that allow to know the simulation values as a function of  $Z$  for  $\sigma_1(Z)$ ;  $\sigma_2(Z)$  and  $E(Z)$ .

### 3.4. Discussion of Results

A priori, it is thought that dark matter WIMPs do not follow the traditional mechanisms of particle interaction with matter. However, all experimentation has embedded the interaction of baryonic matter and dark matter. The pertinent question is: What should we do to separate the two types of interactions? Separating dark matter from barium matter is impossible in our days. A second idea would be to use a 100% reliable mathematical model  $R^2 \rightarrow 1$ , which describes the unique interactions of barium matter in a clear way, in order to compare it with experimental results that logically incorporate dark matter and barium matter. Does dark matter obey the laws of physics applied to protons, electrons and neutrons? The answer is yes, and it can be obtained as subtraction or comparison between the 100% reliable mathematical models (Theoretical or Simulation), as opposed to the experimental results of the same phenomenon. We will demonstrate the demonstration through the variation of the energy and the effective sections between a 100% reliable model of barium matter and the respective experimentation (baryonic matter and dark matter). We will present five experimental evidences and four theoretical evidences, of the presence of dark matter in the

vicinity of the atomic nucleus. Especially, the resonant interaction of dark matter and atomic nucleus for the chromium, Xenon and thulium atoms.

### 3.5. Theoretical Evidences

- Measurement of the cross section of prescient dark matter in the vicinity of the atomic nucleus.
- Measurement of Nuclear Viscosity in alpha decay and concordance with experimental results.
- Magnitude ranges for strong interaction compared to Navier-Stokes strength.

### 3.6. Experimental Evidence

- Variation of energy and effective section between simulation and experimentation only for two elements of the periodic table. From the graphics 5. For the NIST data corresponding to the experimental measurement of the effective sections, while the GEANT4 data are the result of Montecarlo simulation, it is shown that there are two totally differentiable peaks in a range of elements of the periodic table  $11 \leq Z \leq 92$ . This peak corresponds to xenon.
- Defect of mass relevant to thulium and chromium.
- Impact of the presence of dark matter in the vicinity of the atomic nucleus.
- Concordance of magnitude ranges between the gravitational interaction and the strength of Navier-Stokes Nuclear.
- Nuclear Viscosity responsible for nuclear stability [19].

From the theoretical and experimental evidence, it can be inferred that Navier-Stokes force represents gravitational interaction.

### 4. Nuclear Disintegration Evolution

At time  $t = 0$ , before disintegration, the probability equation is stationary.

For example, in case of Uranium and Torium, the probability is  $P = 0.90184$  and concentration  $C = 0.10884 C_0$ .

$$P = \frac{1}{1 + e^{\frac{-6.84025480r}{100}}} = \frac{2}{1 + e^{\frac{-6.84025480r}{100}}}, \text{ the calculation}$$

$$\text{using exact values are } \frac{1}{1 + e^{\frac{-6.8407*3.2422}{10}}} = 0.90184$$

$$\text{and } \frac{2}{1 + e^{\frac{-6.8407*3.2422}{10}}} = 0.90176 \text{ here we note a variation}$$

equal to  $8.8708 \times 10^{-5}$  which is less than 0.001%.

At time  $t = t_{1/2}$ , during natural disintegration

The probability value is  $P = 1/3$  and the parameters can be written as follows:  $k t_{1/2} - \mu r_{1/2} = \ln(2)$  and  $\mu r_{1/2} = 6$ .

On the other hand, the concentration is  $C = \frac{1}{2} C_0$  which holds the correspondence with classical definition.

At time interval  $0 < t < t_{1/2}$ , before the natural disintegration process

The probability evolution interval is given by:  $\frac{1}{3} < P(t, r) < 0.90184$

The concentration evolution interval is given by:  $0.10884 C_0 < C(t, r) < \frac{1}{2} C_0$ .

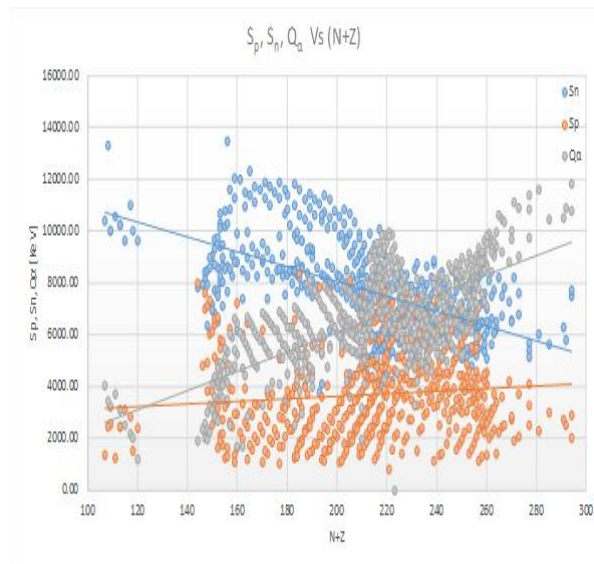
The corresponding figure ( $r(t) \text{ Vs } t$ ) for this process is analyzed when we discuss the results.

### 5. Solution Algorithm

- We define a value of the initial pressure  $p_0$  and of viscosity in atomic nucleus. These variables are determined by instruments with a certain degree of accuracy. However, the measurement of a physical variable always has an uncertainty independent of the accuracy of the instruments. Intrinsic uncertainty is determined by Heisenberg's uncertainty principle.

- The function that solves the Navier-Stokes 3D equation is a logistic probability density (Fermi Dirac probability function). The attenuation coefficient of incident molecules is  $\mu$  which has a positive value, while the coefficient that weighs the evolution in time depends on nuclear pressure and dynamic viscosity  $\frac{p_0}{2\eta}$ . 
$$P(x, y, z, t) = \frac{1}{1 + e^{\frac{p_0}{2\eta}t - \mu r}}$$

(Figures 3).



**Figure 3:** The energy of an alpha particle  $Q_\infty = S_n^a S_p^b$ , is a cooperative equilibrium of neutron energy  $S_n > 0$  and the proton energy  $S_p > 0$  where:  $0 < a < 1$ ;  $0 < b < 1$ ;  $0 < a + b < 1$ .

- For each time value  $t$  there exists a fixed point  $r(x, y, z, t)$  that allows to fully comply with the Navier-Stokes 3D equation. Where  $-1 \leq \frac{x}{r} \leq +1$ ;  $-1 \leq \frac{y}{r} \leq +1$ ; and  $-1 \leq \frac{z}{r} \leq +1$

- If we determine experimentally the value of variation of the parameters  $p_0, v, \mu$ , we find the average value of  $r$  and the standard deviation of  $r$ , determining a spherical cap in which the Navier-Stokes 3D equation are satisfied.

- We calculate probabilities  $P(x, y, z, t)$ , concentrations  $C(x, y, z, t)$ , pressures  $p = p_0 P$  and the velocity field  $\mathbf{u} = -2v \frac{\nabla p}{\rho}$  of fluids. Also, when we apply the respective algorithm, we clearly see how for a definite radius  $r$  that we find an interval with infinite time values

$$t \in \left[ \bar{t} - \left\langle (t - \bar{t})^2 \right\rangle^{1/2} \leq t \leq \bar{t} + \left\langle (t - \bar{t})^2 \right\rangle^{1/2} \right],$$

even for variations of less than 1% in the parameters  $k = \frac{p_0}{2\rho_0 v}, \mu$ . Turbulent flows and

vortices in atomic nucleus explain nuclear instability. The necessary condition for the existence of turbulent flows occurs when the velocity of the nuclear fluid  $|\mathbf{u}| > |\mathbf{u}_e| = \frac{p_0}{2\eta\mu}$  is

greater than the equilibrium velocity  $|\mathbf{u}_e|$ , obtained as a function of the parameters of the medium such as: initial pressure  $p_0$ , dynamic viscosity  $\eta$  and attenuation coefficient  $\mu$ . The sufficient condition for the existence of vortices is given by the fixed point theorem in implicit functions and by the expected value theorem of the logistic density function, which complemented the requirement (6).

- The atomic nucleus, due to its incompressible nature, complies with a rigorous solution of the Navier-Stokes equations. This solution is a generalization of the Fermi Dirac distribution and measures the nuclear stability (**Figures 3**).
- The statistical equilibrium of a physical system such as a uid is obtained for the maximum entropy corresponding to the probability value of  $P = \frac{1}{2}$  and physically it is equivalent to  $kt - \mu r = 0$ .
- The function  $r = \phi(t)$  that appears in the fixed-point theorem and in the expected value theorem needs to be generalized to respect the Heisenberg's theorem, which involves the intrinsic variation of  $x, y, z, t$  and energy, as follows:

$$r = \phi(t, k, \mu) :$$

Stochastic fluctuations of the parameters  $k = \frac{p_0}{2\eta}$  and  $\mu$ .

The evolution of pressure and velocity also depends on the stochastic fluctuations of the parameters, which are local and not global. We obtain a dynamic and probabilistic solution of the Navier-Stokes 3D equation, which represents a fixed point. This function corresponds to a spherical boundary, defined for the solution of the Navier-Stokes equations. This spherical surface is the macroscopic solution domain, and it is a vortex. At the macroscopic level, turbulence centers are also resonance centers where energy is efficiently deposited or captured. Turbulence is where and when all random effects cancel out and only cooperative effects of order and apparent coordination are manifested, creating minimal entropy.

### 5.1. The Tunnel Effect Associated With The Alpha Emission Must Be Revised According To The Navier-Stokes Equations

At the theoretical level we should expect that by the action of the electric and nuclear forces present in

the atomic nucleus, the following inequality in the emissions of protons, neutrons and alpha particles is met.

$$Q_{\alpha} > 2S_p > 2S_n$$

However, the reality is different as we can show for all isotopes with alpha decay and positive energies

$$S_p > 0, Q_{\alpha} > 0, S_n > 0.$$

$$S_p < Q_{\alpha} < S_n, 100 < (N + Z) < 230$$

$$Q_{\alpha} \ll 2S_n, (N + Z) > 231$$

On the other hand, according to electromagnetic considerations, the energy that an alpha particle should have is  $Q_{\alpha} = [\frac{1}{4\pi\epsilon_0}] \frac{2Z\alpha e^2}{R} = \frac{2Z\alpha hc}{r_0(n+Z)^{1/3}} \approx 35$  is equivalent to 35MeV, those after deeper considerations it should be 30 MeV. Reality states that the energy of an alpha particle is in the range  $5MeV < Q_{\alpha} < 9MeV$ : Due to these evidences, the scientific community has assumed that there is a tunnel effect in the emission of alpha particles.

Our results show that the emission of protons, neutrons and alpha particles is produced by a combined effect of centrifugal force and nuclear viscosity and there is no tunnel effect in the alpha emission. In addition, with the considerations established by the different theorems shown here, we can prove with  $R^2 = 0.994$ , the accuracy of our models.

#### ❖ Considerations

- Theorem An action on the nuclear surface produces a reaction in the nuclear volume and vice versa.
- Theorem In the atomic nucleus, the angular moments of the neutron sphere and the proton layer are cancelled.
- Theorem. Nuclear cavitations appears as a function of the solution of Navier Stokes Equations.

#### 5.2. Nuclear Cavitations is Responsible for the Formation of Alpha Particles

The variation of the nuclear pressure forms the alpha particles inside the atomic nucleus. The energy of the protons and neutrons determine the energy of the alpha particles, with a coefficient  $R^2 = 0.994$

.The energy of the alpha particles allows the calculation of the nuclear viscosity.

#### 5.3. Increasing Neutrons is Easier than Increasing Protons in Any Atom

When we increase the number of protons in an atom, the size of the nuclear surface increases immediately due to the force of electrical repulsion. While if we increase the number of neutrons, these are rearranged, even decreasing the size of the atomic nucleus, due to the action of the nuclear force.

Using the previous equations, to find the sensibility of the radius  $R$  in function of  $Z$  and  $N$ , where

$$R^2 = \frac{Z\pi(r_p)^2}{4\pi} \text{ and } R = 1.2N^{1/3} + r_p. \text{ Taking the}$$

derivatives respect to  $Z$  and  $N$ , we calculate the sensibilities of the external radio.

$$\frac{d \ln(R)}{dZ} = \frac{1}{2Z} > \frac{d \ln(R)}{dN} = \frac{1}{3(N + \frac{r_p}{1.2} N^{2/3})}.$$

Taking into consideration that there is a functional relationship between  $Z$  and  $N$ ; using the inequality

$$R \geq 1.2N^{1/3} + r_p \text{ we see clearly } \frac{dR}{dN} = 1.2 \frac{1}{3} N^{-2/3}.$$

#### 5.4. Excess of Angular Momentum

Using Wigner s matrix and the Hamiltonian of the atomic nucleus, we can evaluate the energy for two different geometries of the atomic nucleus and contrast it with the existing theory. In main components, where the moments of inertia of the atomic nucleus are represented by  $I_1, I_2, I_3$ . The Hamiltonian has the form:

$$\hat{H} = \frac{1}{2} \left[ \frac{P_x^2}{I_1} + \frac{P_y^2}{I_2} + \frac{P_z^2}{I_3} \right]$$

For the spherical case  $I_1 = I_2 = I_3 = I$  and for the case of an ellipsoid  $I_1 = I_2 \neq I_3$ . The action of

$P^2 = P_x^2 + P_y^2 + P_z^2$  above Wigner s matrix gives us:

$$P^2 D_{m'm}^j(\alpha, \beta, \gamma)^* = \hbar^2 j(j+1) D_{m'm}^j(\alpha, \beta, \gamma)^*,$$

With a degenerate value of energy  $E_j = \frac{\hbar^2 j(j+1)}{2I}$ . If the protons rotate exclusively around the neutrons,

then the moment of inertia has the form

$$I_p = \frac{2}{5} Z m_p \frac{((1.2 A^{1/3})^5 - (1.2 N^{1/3})^5)}{((1.2 A^{1/3})^3 - (1.2 N^{1/3})^3)}, \text{ whereas if}$$

the protons and neutrons rotate the Moment of inertia changes its value  $I_n = \frac{2}{5} A m_n (1.2 A^{1/3})^2$ .

For heavy atoms the nucleus is deformed  $I_3 < I_1 = I_2$ ; and it takes the form of an ellipsoid.

$$\hat{H} = \frac{1}{2} \left[ \frac{P_x^2}{I_1} + P_z^2 \left[ \frac{1}{I_3} - \frac{1}{I_1} \right] \right]$$

We can also obtain the values of the projections on the z axis, when a magnetic field acts on this axis.

$$P_z^2 D_{mk}^j(\alpha, \beta, \gamma)^* = \hbar^2 k^2 D_{mk}^j(\alpha, \beta, \gamma)^*$$

$$\text{With an energy } E_{jk} = \hbar^2 \frac{j(j+1)}{2I_1} + \hbar^2 k^2 \left[ \frac{1}{2I_3} - \frac{1}{2I_1} \right]$$

## 6. Conclusions

The atomic nucleus is an incompressible fluid, which complies with the Navier-Stokes equations and has two physical layers. The inner sphere formed by  $N$  neutrons and the outer layer formed by  $Z$  protons. The angular moments of the layers are opposite and are cancelled in a classical manner. In addition, it is verified, using Game Theory and Experimental Information, that the alpha particles are formed by cavitation in the nuclear frontier that from the point of view of geometry has analogy with the Bekenstein - Hawking frontier. Finally, black holes have maximum entropy but atomic nucleus has minimal entropy (which equals maximum information). The holographic principle is applicable for general systems, be they quantum or galactic. Therefore, the difference between a black hole that has maximum entropy and an atomic nucleus that has minimal entropy is only the order. The atomic nucleus is constituted by two physical zones that are totally ordered and distinguishable. The layer of  $Z$  protons that form the nuclear surface and the  $N$  - neutron sphere that forms the inner zone. The external surface stores the information of the entire nucleus that has minimum entropy is calculated with the Hawking-Bekenstein equation. For 3D, the holographic principle is nothing more than the application of Gauss's Divergence Theorem, and explains that everything that happens on the surface is equivalent to everything that happens inside (volume).

The nuclear cavitations that completely explains the alpha decay is produced by the atypical variation of the nuclear pressure very well represented by the

solution of the Navier-Stokes equations, which has a probabilistic foundation equivalent to Quantum Mechanics. In addition, the electrical and nuclear forces form an incomprehensible and complex fluid between protons and neutrons. However, this fluid has minimal entropy. With the appearance of complex fluids like the nuclear fluid, we must get used to explaining new phenomena through the Navier-Stokes equations that have the advantage of having a probabilistic foundation in their exact solution, which we have applied creatively. The Information Theory has been enriched by Game Theory. With this mathematical support we have been able to demonstrate that the energy of the alpha particles is formed as a cooperative balance of protons and neutrons represented by the conservation of angular momentum. Moreover, baryonic matter is the result of the collective equilibrium of proton and neutron spins and is a generalization of the known mating. The nuclear and electrical forces contribute 20% to the emission of the helium nuclei, while the force of Navier-Stokes explains 80%. There is already a mechanism to detect dark matter in different parts of the earth using X-ray Femtoscope and the universe using X-ray telescope. The effective location of dark matter detectors using chromium, thulium or xenon in several places on the planet earth and in the space station, must be done under the following premises: Chromium and thulium samples should be installed together with an x-ray equipment to measure the total absorption of photons, ie k-edge, and find the three fundamental parameters: resonance energy  $E(Z)$ , effective section at the beginning of the resonance  $\sigma_1(Z)$  and cross section at the end of the resonance  $\sigma_2(Z)$ . The corresponding theoretical values and corresponding to the Monte Carlo simulation can be obtained with GEANT4. You can find the cross section and the energies of dark matter present in the planet earth. Other characteristics of the behavior and interaction of dark matter can be measured.

## 7. ANEX A1. Navier-Stokes Equation and Cross-section

The speed needs to be defined as  $\mathbf{u} = -2v \frac{\nabla P}{P}$  where  $P(x, y, z, t)$  is the logistic probability function

$$P(x, y, z, t) = \frac{1}{1 + e^{kr - \mu r}}, \quad r = (x^2 + y^2 + z^2)^{1/2}$$

defined in  $((x, y, z) \in \square^3, t \geq 0)$ . This  $P$  is the general solution of the Navier-Stokes 3D equations, which satisfies the conditions (A1)



and (A2), allowing to analyze the dynamics of an incompressible fluid.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho_0} \quad ((x, y, z) \in \square^3, t \geq 0) \quad (A1)$$

Where,  $\mathbf{u} \in \square^3$  an known velocity vector,  $\rho_0$  constant density of fluid and pressure  $p = p_0 P \in \square$

With speed and pressure dependent on  $r$  and  $t$ : We will write the condition of incompressibility as follows.

$$\nabla \cdot \mathbf{u} = 0 \quad ((x, y, z) \in \square^3, t \geq 0) \quad (A2)$$

**Theorem 18:** The velocity of the fluid given by:  $\mathbf{u} = -2v \frac{\nabla P}{P}$ , where  $P(x, y, z, t)$  is the logistic probability function

$$P(x, y, z, t) = \frac{1}{1 + e^{kt - \mu(x^2 + y^2 + z^2)^{1/2}}} \text{ defined in}$$

$((x, y, z) \in \square^3, t \geq 0)$  is the general solution of the Navier Stokes equations, which satisfies conditions (A1) and (A2).

**Proof.** Firstly, we will make the equivalence  $\mathbf{u} = \nabla \theta$  and replace it in equation (A1). Taking into account that  $\nabla \theta$  is irrotational,  $\nabla \times \nabla \theta = 0$ , we have.

$$(\mathbf{u} \cdot \nabla) \mathbf{u} = (\nabla \theta \cdot \nabla) \nabla \theta = \frac{1}{2} \nabla (\nabla \theta \cdot \nabla \theta) - \nabla \theta \times (\nabla \times \nabla \theta) = \frac{1}{2} \nabla (\nabla \theta \cdot \nabla \theta)$$

We can write,

$$\nabla \left( \frac{\partial \theta}{\partial t} + \frac{1}{2} (\nabla \theta \cdot \nabla \theta) \right) = \nabla (-p)$$

It is equivalent to,

$$\frac{\partial \theta}{\partial t} + \frac{1}{2} (\nabla \theta \cdot \nabla \theta) = -\frac{\Delta p}{\rho_0}$$

Where  $\Delta p$  is the difference between the actual pressure  $p$  and certain reference pressure  $p_0$ .

Now, replacing  $\theta = -2v \ln(P)$ , Navier Stokes equation becomes.

$$\frac{\partial P}{\partial t} = \frac{\Delta P}{\rho_0} P \quad (A3)$$

The external force is zero, so that there is only a constant force  $F$  due to the variation of the pressure on a cross section  $\sigma$ . Where  $\sigma$  is the

total cross section of all events that occurs in nuclear the surface including: scattering, absorption, or transformation to another species.

$$F = p \sigma_2 = p \sigma_1$$

$$\Delta p = p - p_0 = \left( \frac{\sigma_1}{\sigma_2} - 1 \right) p_0 = -(1 - P) p_0 \quad (A4)$$

Putting (A3) in (A4) we have

$$\frac{\partial P}{\partial t} = -\mu k (1 - P) P \quad (A5)$$

In order to verify equation (A2),  $\nabla \cdot \mathbf{u} = 0$ ; we

need to obtain  $\nabla r = \left( \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right)$

$$\nabla^2 r = \nabla \cdot \nabla r = \frac{(y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2)}{(x^2 + y^2 + z^2)^{3/2}} = \frac{2}{r}$$

$$\nabla \cdot \mathbf{u} = -2v \nabla \cdot \frac{\nabla P}{P} = -2v \mu \nabla ((1 - P) \nabla r) \quad (A6)$$

Replacing the respective values for the terms:  $r^2 P$  and  $\text{jr} P_j^2$  of equation (A6). The Laplacian of  $P$  can be written as follows.

$$\begin{aligned} \nabla^2 P &= \mu (1 - 2P) \nabla P \cdot \nabla r + \mu (P - P^2) \nabla^2 r \\ &= \mu^2 (1 - 2P) (P - P^2) \left| \nabla r^2 \right| + \mu (P - P^2) \nabla^2 r \\ &= \mu^2 (1 - 2P) (P - P^2) + \mu (P - P^2) \frac{2}{r} \end{aligned} \quad (A7)$$

Using gradient  $\nabla P = \mu (P - P^2) \nabla r$  modulus

$$\left| \nabla P \right|^2 = \mu^2 (P - P^2)^2 \left| \nabla r \right|^2 \text{ and } \nabla^2 P \text{ in (A7)}$$

$$\left[ \frac{\nabla^2 P}{P} - \frac{|\nabla P|^2}{P^2} \right] = 0$$

Replacing equations (A6) and (A7) in (A8) we obtain the main result of the Navier Stokes equations, the solution represents a fixed point of an implicit function  $f(t, r)$  where  $f(t, r) = P - \frac{2}{\mu r} = 0$ .

$$P = \frac{1}{1 + e^{kt - \mu(x^2 + y^2 + z^2)^{1/2}}} = \frac{2}{\mu(x^2 + y^2 + z^2)^{1/2}}, \quad ((x, y, z) \in \square^3, t \geq 0)$$

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### References

1. Kovtun P, Son D, Starinets A (2005) Viscosity in Strongly Interacting Quantum Field Theories from Black Hole. Phys. Rev. Lett 94: 11-25.
2. Susskind L (2009) The Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics. ebook.
3. Marek C (2018) If Gravity is Geometry, is Dark Energy just Arithmetic? Int J Theor Phys 56: 1364-1381.
4. Piotr F (2018) Quantum Approach to Cournot-type Competition. Int J Theor Phys 57: 353-362.
5. Ji-Wan H, Jing-He W, Xian-Ming L (2017) Hawking Radiation via Damour Ruc ni Method in Squashed Charged Rotating Kaluza-Klein Black Holes. Int J Theor Phys, 56, 480-493.
6. Auerbach N, Yeverechyahu A (1975) Nuclear viscosity and widths of giant resonances. Annals of Physics 95: 35-52.
7. Pohl R, Antognini A, Nez F, Amaro FD, Biraben F (2010) The size of the proton. Nature 466: 213-216.
8. Jimenez E, Recalde N, Jimenez Chacon (2017) Extraction of the Proton and Electron Radii from Characteristic Atomic Lines and Entropy Principles. Entropy 19: 293.
9. Aprile E (2017) Search for bosonic super-WIMP interactions with the XENON100 experiment (XENON Collaboration). Phys. Rev. D 96.
10. Conlon J, Day F, Jennings N (2017) Consistency of Hitomi, XMM-Newton and Chandra 3.5 keV data from Perseus. Phys. Rev. D.
11. Simona G (2014) Xray Crystallography: One Century of Nobel Prizes, Journal of Chemical Education 91: 2009-2012.
12. Kozaczuk J (2018) Dark photons from nuclear transitions. PHYSICAL REVIEW D, 97.
13. Kawasaki M, Kohri K., Moroi T (1971) Revisiting big-bang nucleosynthesis constraints on long-lived decaying particles. PHYSICAL REVIEW D, 97.
14. Clay Mathematics Institute (2017) Recuperado el 01 de 05 de 2017.
15. Kulish V, Lage JL (2002) On the Relationship between Fluid Velocity and de Broglie's Wave

Function and the Implications to the Navier Stokes Equation. International Journal of Fluid Mechanics Research 29.

16. Leray J (1934) Sur le mouvement d un liquide visquex emplissent 1 espace. Acta Math J 63: 193-248.
17. Caffarelli, L. (1934). Partial regularity of suitable weak solutions of the Navier Stokes equations. Comm. Pure & Appl. Math, 35.
18. Huang K (1987) Statistical Mechanics (2nd ed.). John Willey & Sons.
19. Geant4 (2016) Guide for Application Developers Version: geant4 10.3.

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